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Three essays on cartel agreements

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Tese apresentada ao Programa de Pós-Graduação em Economia da Universidade Federal de Juiz de Fora, na área de concentração Microeconomia Aplicada, como requisito parcial para obtenção do título de Doutor em Economia.

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Em memória de meu pai, Sérgio, e de meu irmão, Ricardo.

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RESUMO

Esta tese propõe três ensaios sobre acordos de cartel. Assumindo a racionalidade limitada, o primeiro capítulo analisa a interação estratégica entre empresas a partir de uma perspectiva de jogos evolucionários. Nesse sentido, introduz-se novos elementos para capturar e discutir os mecanismos que garantem a estabilidade dos acordos colusivos. No segundo capítulo, usando as motivações econômicas do crime, desenvolve-se um modelo teórico para avaliar a estabilidade dos cartéis ilegais. Portanto, como o cartel age ilegalmente, a punição também se dá nesse âmbito. Assim, apresentam-se novos *insights* para as autoridades antitruste na detecção e inibição de cartéis enquanto organizações criminosas. Por fim, o terceiro capítulo dialoga com os capítulos anteriores por meio de uma avaliação empírica da formação de cartéis no mercado varejista de gasolina nas seguintes cidades: Belo Horizonte, Brasília, Caxias do Sul e São Luís. Combinam-se técnicas de aprendizagem de máquina com filtros baseados nos momentos estatísticos da distribuição de preços de varejo da gasolina para classificar o comportamento do cartel.

Palavras-chave: Cartel. Antitruste. Racionalidade Limitada. Organização Criminosa. Modelos de Simulação por Agente. Aprendizagem de Máquina.

ABSTRACT

In this thesis, we propose three essays on cartel agreements. Assuming bounded rationality, the first chapter analyses the strategic interaction between firms from an evolutionary game perspective. We introduce new elements to discuss the mechanisms that sustain collusive agreements. In the second chapter, following the economic reasoning of crime, we propose a game-theoretical model to evaluate the stability of illegal cartels. Under this approach, punishment is also illegal. Thus, we offer new insights to antitrust authorities in inhibiting cartels as criminal organizations. Finally, the third chapter dialogues with the previous chapters through an empirical assessment of gasoline cartels in Brazil. To reach our purposes, we combine machine learning techniques with screens based on the statistical moments of the gasoline retail price distribution to correctly classify cartel behavior.

Key-words: Cartel. Antitrust. Bounded Rationality. Criminal Organization. Agent-based Models. Machine Learning.

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1 INTRODUCTION

In this thesis, we discuss cartel agreements from different perspectives. Cartel prosecution is becoming a priority policy objective. Increasingly, prohibition against cartels is now considered to be an indispensable part of competition law. Thus, antitrust enforcers should be helped in their ability to detect and avoid cartels by various means and instruments. Therefore, the objective of our study is to provide reasonable answers to the following questions: (a) Does the introduction of altruistic punishment character increase the stability of so-called legal cartels? (b) At what level the retaliation mechanisms adopted by illegal cartels - which act like criminal organizations - are effective in increasing collusion stability? (c) From an empirical point of view, how can we identify behavioral patterns of cartel agreements based on price dynamics? With this in mind, we seek to understand cartel stability using approaches that are still little explored in the Industrial Organization literature. In this way, we open key avenues for improvement of the performance of antitrust enforcers as well as to increase the impact of competition policies.

In order to answer question (a), the first essay evaluates the cartel in a repeated Cournot game. Under the bounded rationality assumption, we aim at focusing on behavioral patterns that sustain the so-called legal cartel agreements. We consider that firms can adopt altruistic-punish behavior, i.e., they are willing to incur extra costs of retaliation which would diminish the expected utility of their payoff to punish firms that deviate from the agreement. This premise opens an avenue to study the cartel with an approach that discusses social organizations and the evolution of cooperation in complex adaptive systems. To evaluate the cartel stability in a dynamic game, we dissociate the punishment strategy from the defection. Thus, we set a game with three strategies. Besides, to capture possible geographic and regional aspects, we restrict competition between firms in a local neighborhood. By introducing an Agent-based model with adaptive learning, our approach enables us to address how often the altruistic punishment behavior can sustain collusion. In short, having considered boundedly-rational firms in the analysis of the deviation strategy and possible retaliation, we provide new insights through a setting that considers a legal cartel.

In order to answer question (b), in the second essay, following the economic modeling of crime developed by Gary Becker, we aim at offering an innovative approach to discuss illegal cartels. This approach deviates from the first essay in the sense that the game is not dynamic and punishment mechanisms are not standard. In other words, we analyze a game in which the cartels act as criminal organizations and the retaliation is illegal. Illegal methods of retaliation can be more harmful to defectors, as they do not have legal remedies to fight back. Hence, this approach brings new insights into how the antitrust authority can inhibit cartel agreements. In this sense, we expand the discussion of the first essay by incorporating the Law & Economics framework. Exploring the economic reasoning of crime, we show that the conclusions regarding the size of stable cartels are incomplete. As well, we offer a useful review of the key aspects of cartel policies, raising methodological issues regarding cartel deterrence.

Concluding, in order to answer question (c), the third essay proposes machine learning screens to evaluate gasoline cartels in Brazil. We selected cities already judged and condemned by the regulator. In this way, we aim at providing empirical pieces of evidence that reinforces the insights brought from the previous chapters as follows. In the gasoline cartels, we have both the dynamic interaction and the illegality aspects, previously addressed in our theoretical game framework. Besides, the regional and local competition features are also relevant, as revealed by the mechanisms of price agreement and retaliation. Finally, we present a general conclusion on the specific contribution of each chapter, summarizing our results as well as establishing some policy prescriptions for the antitrust authority.

2 THE CARTEL AGREEMENT IN AN EVOLUTIONARY GAME PERSPECTIVE: ENFORCEMENT MECHANISM BY A PUNISHMENT STRATEGY

ABSTRACT

In this manuscript, we aim at providing an alternative way of describing the use of the combination of reward and punishment to sustain a cartel agreement through an evolutionary game. In this sense, we evaluate heterogeneous firms' collusive behavior under the bounded rationality assumption. Differently from the traditional approach - based on the Prisoners' Dilemma - we dissociate the punishment from the defection strategy, and punisher firms are willing to give up part of its payoff to punish those firms that betray the cartel agreement. We design a stochastic learning rule through the Agent-based Model (ABM) to capture possible spatial networks effects over agents' strategic behavior. Our findings suggest that: (i) when firms compete in a local neighborhood, the effectiveness of punishment is lower than when compared to the competition in a well-mixed grid; (ii) in most cases, the co-existence of cooperators and defectors is the dynamic balance of the game.

keywords: Cartel agreement. bounded rationality. social dilemma.
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2.1 INTRODUCTION

Due to the incipient contribution of the traditional economic literature to the recognition of some behavioral and strategic patterns that encourage cartel settlements, the general objective of this paper is to provide a new framework for understanding the punishment and reward mechanisms that sustain firms' collusive behavior. To achieve our goal, we assume that firms are boundedly-rational (ELLISON, 2006; SILVEIRA; VASCONCELOS, 2019).

In other words, to sustain the cartel, we consider that firms can adopt altruistic-punish behavior, insofar as they are willing to incur extra costs of retaliation, which would diminish the expected utility of their payoff, to punish firms that deviate from the agreement. This premise opens an avenue to study the cartel with an approach that discusses social organizations and the evolution of cooperation in complex adaptive systems (CHAN et al., 2013; ROCHA, 2017). Therefore, one innovation of our approach is to introduce new theoretical elements. As well, we propose methodological instruments that are still little explored in the Theory of Industrial Organization in the analysis of the market structure and in the description of how the use of reward and punishment can induce and sustain cartel arrangements.

In Industrial Organization (hereafter, IO), cartels are the unlawful agreements between firms to fix prices or quotas of production and division of markets. Typically, coordinated action among firms to eliminate competition and raise product prices occurs in oligopolistic markets where a small number of profit maximizer firms are producing homogeneous goods (TIROLE, 1988). Besides, as in a monopoly, firms under cartel agreements aim to maximize profits. Thus, it is incongruous to think that illegal actions are legitimized by economic theory.

In this way, the illegality of the cartel is confused with that of the monopoly simply because monopoly profits are as high as possible in the market without effective competition. However, it would be more realistic to think that the cartel has an objective function that not only absorbs the monopoly's pricing behavior but also assimilates the pros and cons of anti-competitive behavior. That is, the objective function of the cartelized firms should include costs related to illegal activities, such as punishments in the event of conviction by the antitrust authorities and the punishment of potential whistleblowers (BELLEFLAMME; PEITZ, 2010).

Hence, starting from a motivation supported by the economic theory (profit maximization), firms adopts a collusive (and illegitimate) market behavior, whose characteristics are similar to a particular situation of imperfect competition (monopoly), in which a single firm owns the market for a particular product or service, and is, therefore, able to influence the price of the good. In addition to such behavior, we usually observe a loss of consumer welfare. Furthermore, considering very rare exceptions, such as the OPEC cartel, collusion is typically considered to be unlawful (SPAGNOLO, 2004).

Thus, understanding the mechanisms of cooperation, in this case, is only intended to provide elements for the formulation of policies that will increase the internal instability of the cartel. Often, among individual profits maximizers, an infinite number of strategic interactions are required to achieve cooperative behavior. The hypothesis that the individual is endowed with full rationality, motivated basically by the relation between the cost and the benefit of following or not a set of social norms, aiming the maximization of its economic reward is the basis of the classic Game Theory. It is widely applied in cartel agreement analysis together with a representation of a social dilemma named as Prisoner's Dilemma (PD) (NEUMANN; MORGENSTERN, 2007).

Informally, a social dilemma represents a conflict between the individual and collective interests. It illustrates situations in which individual benefits from selfishness unless everyone chooses the selfish alternative, in which case the whole group loses. Conflicts arise when too many group members choose to pursue individual profit and immediate satisfaction rather than behave in the group's best long-term interests. Social dilemmas can take many forms and are studied across disciplines such as economics, political science, psychology, physics, and biology. Formally, the PD[1]. is widely used in Game Theory to describe such situations. The cardinal property of its equilibrium outcome is individually rational - in the sense that no individual has a unilateral incentive to change one's strategic behavior - and collectively irrational - insofar as the coordination of collective strategic behavior via cooperation could lead all individuals to a Pareto superior situation (AXELROD, 1997; KOLLOCK, 1998).

For Bowles and Gintis (2011), since individual incentives and collective interests are conflicting, whenever cooperation between individuals has a cost, there is a possibility of observing an opportunistic (free rider) behavior at the expense of others' efforts. This makes it harder to get better social (collective) outcomes. A wide literature[2] is devoted to the study of ways in which cooperation could emerge in social dilemmas. In Axelrod and Hamilton (1981), the basis of cooperation in social dilemmas is beyond a choice with a short-run cost and a possible long-run benefit. It includes collaboration with others to build and enforce norms of conduct (not necessarily legal), to impose an industrial standard, to build a new organization that can act on behalf of its members, such as in

[1] Besides the Prisoner's Dilemma, the literature on social dilemmas has developed around different metaphorical stories, such as Public Goods Game and Tragedy of the Commons (EATON; ESWARAN, 2002; CAPRARO, 2013)

[2] Friedman (1971) seminal article shows how sufficiently patient agents cooperate in an infinite Prisoner Dilemma game.Bó (2005), Duffy and Ochs (2009) also provide evidence in this direction. Bó and Fréchette (2011), Gallice and Monzon (2017) proposes mechanisms to sustain cooperation in an environment of uncertainty through experimental studies in infinitely repeated games.

cartel agreements.

Assuming full rationality, the main literature about collusion deal with two different approaches. The first one considers the explicit pricing behavior of firms in an infinitely repeated game. Explicit price agreements are prohibited by law and firms adopt less obvious methods to manage and coordinate their pricing strategies. This leads us to the second approach, which is related to situations where coordination of prices takes place in an implicit way, known as tacit collusion. Firms use a discount factor for expected future profits to evaluate the reward of whether or not to adhere to the tacit agreement. Therefore, the discount factor is a crucial rule in determining the stability of the cartel, i.e., stable collusion is only possible if cartel members attach a sufficiently high value to their future earnings[3].

Following Bernheim and Madsen (2017), the analysis obtained through strategic models of repeated interactions have brought valuable information, but also left an important gap[4] to be filled in. Thus, while substantial progress has been made in formulating cartel theories that respond to a variety of empirically tested standards, some questions remain open. Typically, cartels maintain agreement stability by subjecting participants to punishment[5], but there is room for a better explanation of two important factual observations: first, deliberated deviations from the agreement occur; second, even in the face of defection, deviant firms might not be punished - even if detected. Thus, the traditional approach is also unsatisfactory in the way it draws the punishment to achieve the collusive outcome.

As the existing literature about collusive agreements does not adequately map such events, theories of imperfect information, such as Green and Porter (1984), were formulated to provide explanations of the reasons why cartels tend to disintegrate, giving rise to price wars and retaliation strategies by firms [6]. This line of research attributes the

[3] As exposed in Symeonidis (2002), Levenstein and Suslow (2006), Bruttel (2009), there are several experiments on Bertrand as well as Cournot competition, considering various market design variables concerning their influence on stability of collusive behavior. When analyzing the impact of all those factors most of them implicitly presume that the critical discount factor δ^* comprises a measure for the stability of cooperative behavior in the market. In theory, however, the critical discount factor should only matter for firms behavior in so far as collusion is a sustainable outcome when their actual discount factor δ is larger than the critical δ^*, but not when it is smaller. Thus, we can derive a minimum discount factor above which collusion can be sustained in a subgame perfect equilibrium.

[4] For empirical and theoretical papers on pricing and cartel, see Green and Porter (1984), Rotemberg and Saloner (1986), Abreu, Pearce and Stacchetti (1986), Abreu (1988), Bernheim and Whinston (1990), Athey and Bagwell (2001), Athey, Bagwell and Sanchirico (2004), Athey and Bagwell (2008), Harrington and Skrzypacz (2011).

[5] As demonstrated by Spagnolo (2004), the cartel organization can increase the internal punishment, i.e., the cost of cheating to favor its internal stability.

[6] This issue has been widely discussed in the literature, highlighting Green and Porter (1984), Genesove and Mullin (1998), Genesove and Mullin (2001), Marshall et al. (2016).

collapse of pricing exclusively to exogenous events, that is, that are beyond the control of cartel members - and not to their intentional choices. This implies that cartel members will never deliberately betray collusive agreements. Moreover, according to these theories, if cheating occurs and is detected, the punishment would be an immediate consequence.

With this in mind, it is important to point that, as presented in Camerer (2011), there is a reductionism of the economic agent being seen merely as a utility maximizer once it does not take into account the behavioral, cognitive and emotional aspects inherent to the decision-making process. In other words, in the economic relations of everyday life, people act according to intrinsic motivations and behave according to the ethical standards of society. This often goes against the purely economic and utilitarian interests of the individuals, as explained by Lambsdorff (2007). Coricelli, Rusconi and Villeval (2014) states that in many economic decisions, the so-called non-economic[7] motivations, such as altruistic behavior or ideological activism, can exert dominance over the economic motivations. Thus, rationality assumptions have been widely debated.

Many contributions[8] come from the Evolutionary Game Theory (EGT), which, based on the premise of bounded rationality of agents, enriched the analysis of situations represented by dilemmas - whether social or from other dimensions - as presented in Friedman (1991), Hauert and Doebeli (2004). Besides that, the EGT framework takes into account, aspects such as the existence of biases and heuristics[9] that can affect the decision making process; the dimension of reciprocity[10] and the learning models in complex systems with Agent-Based Simulation (ABS) models.

According to Smith and Price (1973), in EGT, convergence to the dominant long-run equilibrium is expected. In this equilibrium, achieved after a period of dynamic interaction, players must have adopted an evolutionary stable strategy (ESS), which is a strategy in which players have no incentive to abandon unless some external force disturbs the underlying conditions of the game. Then, if classical game theory can be defined as the science that studies strategic behavior, with the theory of evolutionary games it takes a step forward since we now have the science that studies the robustness of strategic behavior.

With all this motivation in mind, we aim to reach the following specific objectives:

[7] In the sense that it leads the agent to a non-optimal (maximum) result of its utility function.

[8] On this matter, Ellison (2006) states that EGT models remedies some drawbacks of the traditional game theory and, recently, has been largely applied in IO topics. Please see Young (1993), Binmore, Samuelson and Young (2003), Cabrales and Serrano (2011), Weidenholzer (2012).

[9] Defined in Camerer (2011) as cognitive processes employed in partial rationality decision making. The strategy ignores part of the information to make adaptive choices in real environments.

[10] Tremblay and Tremblay (2005), Spiegler (2011) address reciprocity in the sense that people are willing to punish devious behavior by seeking a fair and reciprocal treatment (fairness).

(a) to identify the nature of the stability of the cartel agreement; (b) to understand the dynamic pattern of the strategic interaction among firms as it approaches the equilibrium; (c) to estimate the frequency of the punish character which enforces the cartel agreement; (d) to check if stability of the cartel depends on the initial condition of the game, and, if so, how? The results suggest that in some specific cases, punishment is effective in eliminating the cheaters of the cartel agreement. In many other cases, the co-existence of cooperators and defectors is the balance of the game.

To reach our purpose, Section 2.2 introduces the model. Section 2.3 presents the ABS algorithm to assess the effectiveness of punishment in sustaining the cartel. Section 2.4 concludes and discusses further research possibilities.

2.2 THE GAME MODEL

Suppose a linear $n-$firm Cournot model with constant and identical marginal costs of productions. Let n, be the number of firms that produce a homogeneous good with marginal cost mc. The inverse demand function is given by $P(q) = a - q$.

Considering the possibility of collusion and assuming the existence of a single Nash equilibrium in pure strategy in the Cournot Game, the single optimal quantity produced by the cartel is given by $q^M = \arg\max q(P(q) - mc)$. The factor $q(P(q) - mc)$ is monotonically increasing until q^M and, then, it is monotonically decreasing.

The cartel payoff is denoted by $\pi_i^M = q^M(P(q^M) - mc)$. If $q_i = q^M/n$ for all firm i in the stage game, then each firm earns a cartel payoff $\pi^M = \pi^M/n$. When one firm deviates, it earns $\pi_i^D > \pi_i^M$. When firms compete, they earn the oligopoly payoff π_i^O. Thus, $\pi_i^D > \pi_i^M > \pi_i^O$. Typically, as in the repeated Prisoner's Dilemma, the Nash equilibrium reveals that the cartel is not stable, as firms mutually defect and therefore receive π_i^O.

2.2.1 The prisoner's dilemma revisited

The Prisoner's Dilemma is the starting point for bringing the essential elements for understanding the evolution of cooperative behavior in non-cooperative games. It illustrates that cooperating individuals are prone to exploitation, and that dynamic interaction should favor cheaters (or defectors). In this game, two players simultaneously decide whether to cooperate (C) or defect (D). Cooperation results in a benefit b to the recipient but incurs a cost k to the donor. The model assumes that $b > k > 0$. Within our discussion of cartels, firms analyze the cost-benefit of colluding.

Costs can be divided in (i) expenses involved in maintaining the agreement, such as monitoring and meetings, given by k_M; (ii) the opportunity cost that firms incur by choosing not to deviate from the agreement and, therefore, to gain a larger share of the market, given by k_O. Mutual cooperation (C, C) thus pays a net benefit of $\pi_i^M = b - k$, where $k = k_M + k_O$. Mutual defection (D, D) results in the oligopoly payoff for both players, which from now on we normalize to zero $\pi_i^O = 0$. With unilateral cooperation, defection (D, C) yields the highest payoff, $\pi_i^D = b$, at the expense of the cooperator (C, D) bearing the cost $\pi_i^S = -k$. It follows that it is best to defect regardless of the co-players decision. Thus, from the payoff matrix (2.1), defection is a dominant strategy, even though all individuals would be better off if they all cooperated. This outcome is a consequence of $\pi_i^D > \pi_i^M > \pi_i^O > \pi_i^S$.

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} b-k; b-k & -k; b \\ b; -k & \mathbf{0; 0} \end{pmatrix} \end{array} \tag{2.1}$$

Despite this argument seems quite convincing and widely used in cartel analysis, Axelrod and Hamilton (1981), Bowles and Gintis (2011) highlights that, in the evolutionary game framework, it is possible to observe altruistic behavior (i.e., individuals bears costs to the benefit of others) in many situations related to cooperation. Infield and experimental studies it is often difficult to assess the expected payoffs for different behavioral patterns, and even the proper ranking of the payoffs is challenging. This has led to a considerable gap between theory and experimental evidence, and to an increasing questioning with the Prisoner's Dilemma as the only model to discuss cooperative behavior. Following this thought, we will propose an alternative approach to evaluate the stability of cartel agreements in the following subsection.

2.2.2 Enforcement mechanism by a punishment strategy

Thus, for a cartel to survive in the market, credible punishment should be in place to penalize members that defect and, therefore, sustain the cartel agreement (GREEN; PORTER, 1984; JASPERS, 2017). As exposed in Jr and Chen (2006), there are many forms of credible punishments related to price-cutting and the threat of price wars. In short, the traditional economic literature approach seeks explanations for cartel stability through effective internal detection and punishment. This introduces the expectation that the cases will demonstrate sophisticated systems of coordination, monitoring, and enforcement. Retaliation in the form of price slicing and price wars will serve to increase the costs of cheating, thus ultimately stabilizing cartels.

On the other hand, recent empirical studies, such as Levenstein and Suslow (2006), Harrington and Chang (2009), Levenstein and Suslow (2011), states that: (a) cartels invest more in ways to avoid cheating than to resort to ex-post punishments, which are costly; (b) retaliatory response to the defectors increases the likelihood of a cartel's natural demise. In this sense, the deviating effects of internal punishments leave room for alternative explanations of the long-term stability of cartels. Furthermore, from the perspective of a social dilemma, the assumptions about the agent's behavior, proposed by the standard economic literature, may not be sufficient to understand the determinants of cartel success. With this in mind, we focus on studies related to the evolution of cooperative behavior in social dilemmas to design an enforcement mechanism by a punishment character.

As shown in Fehr and Gachter (2000), Xu et al. (2011), punishment has a lead role in promoting cooperation in social organizations within complex adaptive systems. Following this framework, we introduce a third punishing (P) character on our so-called Cartel Agreement Game with boundedly-rational firms and study its effects.

Consider first the polar case in which the boundedly-rational firms are homogeneous. Assume that when firms cooperate (C, C) they both earn the cartel profit and split the cost k, earning $\pi_i^M - k/2$. Unilateral cooperation (D, C) yields π_i^D to the defector. Unlike the PD, the cooperator (C, D) expected payoff is $\pi_i^M - k$. Note that in this situation, the cooperator acquires the benefit of the cartel, but incurs the entire cost k.

The punisher firm carries basically a cooperative (C) character, and are willing to incur in a extra cost χ, in order to punish a defector firm (D) by an amount represented by γ, with $\gamma > \chi > 0$. By doing so, the loss incurred by firms that defect from the cartel is larger than the cost paid by firms that act as punishers, i.e., the underlying concept of bounded rationality is still preserved.

Besides, as we have added a punishment strategy, the two-player game is now 3x3 instead of 2x2. We represent the expected costs and benefits of such altruistic and punitive behavior in the payoff matrix (2.2). Note that it subtly change the relationship between cooperation and defection as well as the ranking of the payoffs as shown (2.1).

$$\begin{array}{c} \\ C \\ D \\ P \end{array} \begin{array}{c} \begin{array}{ccc} C & \quad D & \quad P \end{array} \\ \begin{pmatrix} \pi_i^M - \frac{k}{2} & \pi_i^M - k & \pi_i^M - \frac{k}{2} \\ \pi_i^D & 0 & \pi_i^D - \gamma \\ \pi_i^M - \frac{k}{2} & \pi_i^M - k - \chi & \pi_i^M - \frac{k}{2} \end{pmatrix} \end{array}. \tag{2.2}$$

Observe that whenever a punisher and a cooperator meet, it represents the same situation as if two cooperators were meeting. For simplicity, let us consider the situation in which $\pi_i^D = \pi_i^M > k > 0$. By this assumption, we simplify the analysis, to give greater emphasis on the cost to the benefit ratio when both firms are willing to join the cartel agreement, $\rho = (k/2\pi_i^M - k)$, with $\rho \in (0, 1)$. Thus, considering homogeneous firms, the payoff matrix[11] presented in (2.2) is now given by the normalized payoff matrix (2.3)

$$\begin{array}{c} \\ C \\ D \\ P \end{array} \begin{array}{c} \begin{array}{ccc} C & D & P \end{array} \\ \begin{pmatrix} 1 & 1-\rho & 1 \\ 1+\rho & 0 & 1+\rho-\gamma \\ 1 & 1-\rho-\chi & 1 \end{pmatrix} \end{array}. \tag{2.3}$$

Now, we introduce an asymmetry between firms regarding their behavior towards the cost by the benefit of cooperation, ρ. On this matter, we keep our analysis on the linear $n-$firm Cournot model, but now we assume non-identical marginal costs of productions. We now have a two-population game, where low-cost (Type A) firms are competing with high-cost (Type B) firms.

Namely, Type A firms are more efficient in the sense that they have a lower cost of production and lower cost of monitoring and organizing of the cartel agreement. This makes it possible for them to require a lower ρ than Type B firms. The latter are less efficient and have higher costs and, therefore, require a higher ρ to cooperate with the cartel. So, we have a situation in which $\rho_A < \rho_B$, i.e., Type A firms are more willing to cooperate than Type B firms.

In this regard, the punisher from population A incurs in an extra cost χ_A to punish the defector from population B by an amount γ_B. This reasoning is analog for punisher firms from the population B. To better capture this heterogeneity among players, we will consider that Type A firms will evaluate the possibility of making the collusive agreement with Type B firms and vice versa. In our model, there is no interaction between firms of the same Type, i.e., own-population effects are not taken into account in the two-population game.

In this sense, let us consider that the stage game as represented in (4.4) is repeatedly played over time between two firms, each belonging to one of two very large different populations of firms from Type A and Type B.

$$\begin{array}{c} \\ C \\ D \\ P \end{array} \begin{array}{c} \begin{array}{ccc} C & D & P \end{array} \\ \begin{pmatrix} 1;1 & 1-\rho_A;1+\rho_B & 1;1 \\ 1+\rho_A;1-\rho_B & 0;0 & 1+\rho_A-\gamma_A;1-\rho_B-\chi_B \\ 1;1 & 1-\rho_A-\chi_A;1+\rho_B-\gamma_B & 1;1 \end{pmatrix} \end{array}. \tag{2.4}$$

Now, we can evaluate the effectiveness of the punishment strategy related to: **(a)** the stability of the cartel agreement as a function of ρ, χ and γ; **(b)** the dynamic pattern of the strategic interaction among firms as it approaches the ESS; **(c)** the level of punishment that enforces the cartel agreement: **(d)** the sensibility of the collusive behavior to the

[11] In order to get the payoffs as a function of ρ, it is necessary to multiply all entries in matrix (2.2) by $\frac{2}{2\pi_i^M - k}$:

initial conditions of the game. Given the number of strategies, as well as the number of parameters inserted in our analysis to capture effects arising from the interaction between heterogeneous firms, these questions might be better answered and understood numerically than analytically. Thus, following the vast literature that is dedicated to the analysis of social dilemmas and complex systems through numerical simulations, as Axelrod (1997), Hauert and Doebeli (2004), Nowak and Highfield (2011), Xu et al. (2011), Chan et al. (2013), Rocha (2017), in the next section, we introduce our ABS Model.

2.3 AGENT-BASED SIMULATION MODEL

A stochastic component is implemented[12] in the analysis of evolutionary equilibria using the ABS method, which has been largely used in the understanding of the evolution of cooperative behavior. For Eaton (2004), an oligopolistic market in which firms compete in price or quantity can be understood as a form of social dilemma[13]. In this section, an algorithm to complement the evolutionary game and to guide the dynamic interaction among firms is presented. We consider the competition in which the two populations are distributed in a well-mixed and in a spatial-structured network. A well-mixed arrangement may be consistent with online e-commerce marketplaces, where the geographical position of firms does not directly impact competitors' strategic decisions. A game played in a spatial network can be consistent with a physical and regional market competition, for example: between supermarkets or gas stations in a specific neighborhood or region.

2.3.1 Well-mixed two-population game

To implement the computational simulation in the well-mixed two-population game, the framework presented by Rocha (2017) is followed. In this sense, at a time $t = 0$, we establish an initial proportion of firms' A and B in each population, (f_{A_c}, f_{B_c}), that play the (C) strategy. Evolutionary dynamics are introduced in sequence and, at each Monte Carlo time Step (MCS), a Focal Agent i, which can update[14] its strategy, is chosen randomly. This occurs simultaneously in both populations. Focal Agent i, in turn, plays against a random opponent from the rival population and starts the game presented in matrix (4.4). Thus, Focal Agent i will obtain a payoff V_i. At the same time, an agent j is randomly chosen as a Reference Agent, which randomly plays against an opponent from the rival population and obtains a payoff V_j. Notice that focal and reference players

[12] To implement the algorithm, we made use of the Java programming language.
[13] Other works in this line can be seen in d'Aspremont and Jacquemin (1988), Goodwin and Mestelman (2010), Potters and Suetens (2013)
[14] Two mechanisms to update the population regarding the strategic interactions are largely used in agent-based simulation methods: synchronous and asynchronous. Here, we use the second, since it allows the overlapping generations interactions. See Hauert (2002), Chan et al. (2013).

belong to the same population. The Focal Agent i compares V_i and V_j to analyze the possibility of updating his strategy in two stages as stated ahead: (i) If $V_i \geq V_j$, focal player keeps his strategy; (ii) If $V_i < V_j$, focal player might update his strategy to the one adopted by reference player with a probability given by the variable w:

$$w = \frac{V_j - V_i}{max.payoff - min.payoff} \tag{2.5}$$

The maximum and minimum payoffs are obtained from the game matrix. By doing this procedure, we guarantee that $w \in (0, 1)$. To establish a decision criterion whether Focal Agent i updates his strategy or not, a random number generator is used and is conveniently named $rnd \in (0, 1)$. In this way, a stochastic component on the dynamics of the game is implemented. Focal Agent i compares rnd with the probability w, so that: (iii) If $w \geq rnd$, Focal Agent i updates its strategy and imitates the Reference Agent j; (iv) If $w < rnd$, Focal Agent i does not update its strategy.

At every MCS, randomly selected individuals from both populations have the opportunity to, on average, change strategy at least once, comparing their payoffs with the Reference Agent j. We say that, on average, individuals can update strategies once, because within an MCS the same player may be invited to play many times, and other players may not, since the process of players' selection is random. Thus, when all players in both populations, on average, have the opportunity to update their strategies, an MCS is completed and a new MCS starts to repeat the dynamics of the game. This procedure characterizes the ABS model.

2.3.2 Spatial-structured two-population game

When applying this procedure in regular lattices, the number of opponents with which individuals interact depends on the spatial arrangement of the game and directly impacts the value of w. It is important to notice that, in the well-mixed two-population design all players are arranged in a N-dimensional vector and can compete with the whole rival population, i.e., the probability of interaction between them is the same and independent of their position in the vector. In a spatial-structured network, the concept of local neighborhood emerges, and there only will be competition among players that belong to certain positions in the $N \times N$ dimensional matrix.

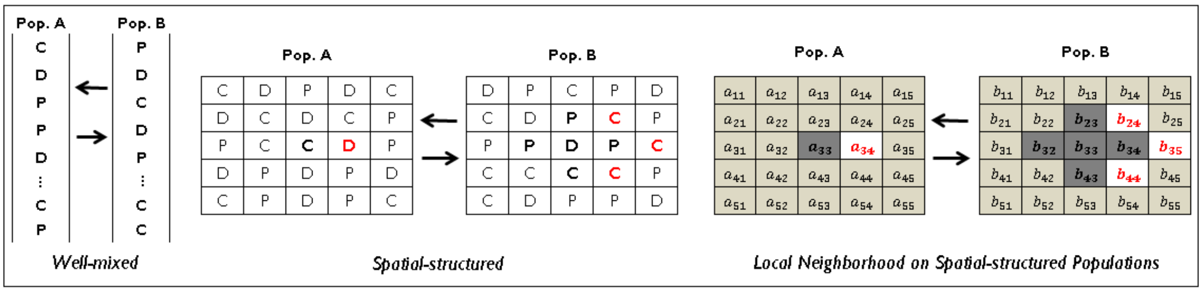
Figure 1 presents an illustration of the two-dimensional game dynamics in a well-mixed and spatial-structured format. In this last is possible to identify the neighborhood of each focal and reference agent according to their position in the matrix. In this paper, we consider the von-Neuman[15] neighborhood. Suppose that the element $\left\{a_{33}\right\}$, illustrated on the right side of Figure 1, is the focal agent that is programmed to play the strategy C.

[15] That consists of four cells arranged orthogonally around the central cell. For detail, see Hauert and Doebeli (2004), Nowak and Highfield (2011). We chose this neighborhood for

According to the von-Neuman neighborhood, the reference agent of $\left\{a_{33}\right\}$, i.e., the one which he will compare his payoff, to decide whether he updates his strategy or not, could be randomly selected between the elements $\left\{a_{34}, a_{32}, a_{23}, a_{43}\right\}$. Suppose, for example, that $\left\{a_{34}\right\}$, who is programmed to play D, is selected as the reference agent.

The elements of the rival population B, with which the focal agent $\left\{a_{33}\right\}$ competes are $\left\{b_{33}, b_{23}, b_{43}, b_{34}, b_{32}\right\}$. The reference agent $\left\{a_{34}\right\}$ competes with the individuals located at $\left\{b_{34}, b_{33}, b_{35}, b_{24}, b_{44}\right\}$. The same interaction happens simultaneously in the opposite direction, that is, there is competition between players of population B with the population A and vice verse. Attention should be paid in considering the strategic interactions of those players located on the border of the spatial structure. The local neighbor with which element $\left\{a_{11}\right\}$ randomly selects to compare his payoff is one of the elements $\left\{a_{51}, a_{21}, a_{15}, a_{12}\right\}$ and the set of players from the rival population that $\left\{a_{11}\right\}$ compete are located in the cells $\left\{b_{11}, b_{12}, b_{15}, b_{21}, b_{51}\right\}$. In this matter, we can see the spatial structure similar to a toroid[16].

Figure 1 – Game dynamics in a well-mixed and spatial-structured populations.



Source:Elaborated by the authors.

The strategy update criterion is calculated by the average payoff of each player, founded by the arithmetic mean of the payments (V) obtained in each interaction with the n players that compose the local neighborhood. The Focal Agent i and the Reference Agent j now receive a payoff given by, respectively:

$$V_i = \frac{\sum_{i=1}^{n} v_i}{n}, \quad V_j = \frac{\sum_{j=1}^{n} v_j}{n} \tag{2.6}$$

As explained before, if $V_j > V_i$, the focal agent may imitate reference agents' strategic behavior with probability w. Note that w is based on the averages received by

its better dialogue with the economic literature on collusion, specifically with the paper of Selten (1973), entitled "A simple model of imperfect competition where four are few and six are many".

16 In mathematics, a toroid is a surface of revolution with a hole in the middle, like a doughnut, forming a solid body. The axis of revolution passes through the hole and so does not intersect the surface.

the focal and reference agents. The worse the focal players' performance in relation to the reference, the greater the probability that he imitates reference agents' strategy.

2.3.3 Results

In this subsection, we present the results obtained with the ABS algorithm. First, we show the effect of punishment for the well-mixed two-population game and evaluate its effect on the level of cooperation with the cartel among firms. To capture some possible peculiarity regarding the effect of spatial competition, in which firms interact only with their local neighbors, we evaluate the role of punishment applied to the deviant firms of the collusive agreement in a spatial-structured two-population competitive arrangement. It is important to report that the simulations were carried out a substantial number of times to capture the random effects and the graphs presented here refer to the average of the typical behavior of the firms.

To generate the results, the simulations were performed considering the game presented in (4.4). We firstly assume that $(\chi_A, \gamma_B) = (0.1; 0.5)$ and $(\chi_B, \gamma_A) = (0.01; 0.05)$. In other words, in the numerical simulation performed, the punishment suffered by defectors (γ_{-i}) from the rival population is five times greater than the cost of punishment (χ_i). Then, we tested different combinations for the values of ρ_i, keeping χ_i and γ_i constants, to infer how the cost of cooperation with the cartel agreement impact firms strategic decision and the dynamic balance of the game. The values of the parameters respect the restrictions imposed in the elaboration of the payoff matrix.

Thus, the population dimension[17] of each firm is set $N_A = N_B = 8.100$, and the initial proportion of firms programmed to adopt each of the pure strategies available in the game is given by $(f_{C_A}, f_{D_A}, f_{P_A}) = (0.1; 0.1; 0.8)$ and $(f_{C_B}, f_{D_B}, f_{P_B}) = (0.1; 0.8; 0.1)$. These initial conditions are intended to reflect the greater willingness to collude of Type A firms. In other words, they are more willing to punish firms from the rival population so that the cartel remains stable. In turn, Type B firms, which have a higher cost for the benefit of cooperation, is more likely to defect from the agreement and benefit from the deviation.

[17] Many population dimensions, both smaller and larger were also tested and did not change the results presented here. The only variation was the speed of convergence to the steady state. Besides, for instance, this number is under the amount of players that exist in a digital economy environment (well-mixed) and with the amount of gas stations or supermarkets in big urban centers (spatial-structured). Note that in spatial-structured competition, although there are many players, the relevant market is determined by the local neighborhood, that is, composed only by 5 firms. In this way, we ensure the robustness of the simulations without compromising the theoretical structure of the oligopoly and the cartel agreements.

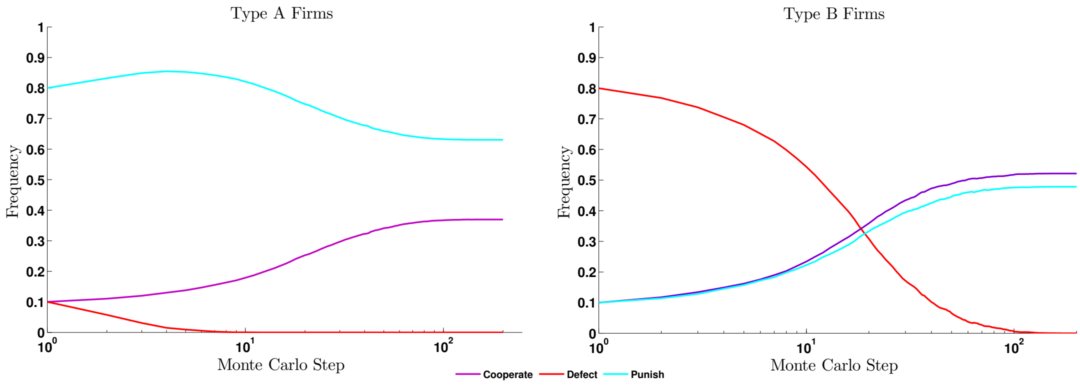
2.3.4 Well-mixed competition

In this arrangement, the probability of strategic interaction among firms do not depend on their location in the grid. We set $\rho_A = 0.01$ and $0.01 < \rho_B \lesssim 0.27$ and achieved two distinct steady states. In the first, illustrated in figure 2, the steady state is formed by the co-existence of firms that cooperate with the cartel agreement and firms that punish those that defect. Thus, the punishment is effective in eliminating the deviant behavior of the market.

On the other hand, figure 3 shows a steady state that is characterized with all Type A firms cooperating and all Type B firms defecting from the collusion. This last result shows the difference between the willingness to cooperate in each population. As Type B firms require a higher ρ, once this value increases, the greater the benefit gained from diverting from the agreement. For situations where: (a) we shorten the distance between ρ_A and ρ_B, by assigning $\rho_A > 0.01$ and $\rho_B \leq 0.27$ and; (b) we vary this interval, by setting $\rho_A \geq 0.01$ and $\rho_B > 0.27$, we observed that the steady state is reached with Type A firms cooperating and Type B firms defecting at the expense of the rival population. In this way, although the punishers can eliminate defectors, this balance is not unique. Thus, since Type B firms require a ρ_B rate much larger than ρ_A, the observed net effect is that, since Type A firms signal intent to participate in the agreement, Type B firms acquires a greater payoff when defecting.

Note that the game dynamics in the initial MCS of figures 2 and 3 are quite similar. Between $0 < MCS \lesssim 10$ we observe the complete elimination of the defectors of the cartel agreement in population of Type A firms, who update their strategy to C or P. Simultaneously, we see that around $MCS \approx 20$, in population of Type B firms, the relative frequency of defectors becomes lower than that of cooperators and punishers. This suggests that the punishment applied by Type A firms at this stage of the game had a positive effect over Type B firms. The reverse movement is also true. The net effect is an increasing in the relative frequency of punishing and cooperating firms in both populations.

Figure 2 – The co-existence of C and P characters in both populations.

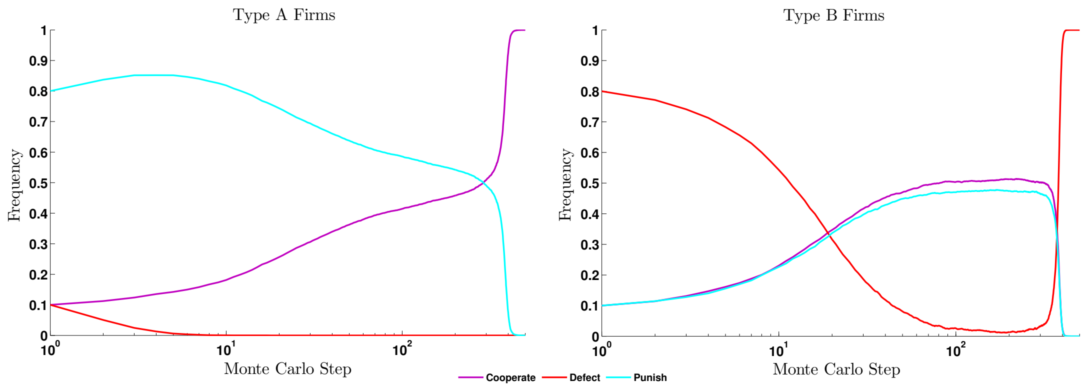


Source: Elaborated by the authors.

What differentiates the steady states that will set the game balance can be observed between $100 \lesssim MCS \lesssim 150$. Note that in Figure 2, the frequency of punishers in the population of Type A firm stabilizes above the frequency of cooperators, that is, $f_{P_A} \approx 0.63$. Due to this, firms of the rival population that are playing D, sensing the effect of punishment, update their strategic behavior and start to cooperate with the cartel agreement. Therefore, we have reached the result in which cartel defectors are completely eliminated from the market.

On the other hand, in order to explain the result presented in 3 we observe that around $100 \lesssim MCS \lesssim 150$, the frequency of punishers in the population of Type A firm becomes lower than the fraction of cooperators, i.e., $f_{C_A} > 0.5$. Due to this, the frequency of deserters of the rival population grows at the expense of Type A cooperating firms. Then, given that there is an increasing number of defectors in the population of Type B firms, the best answer for Type A firms is to play C, and not P once the punishment leads to a payoff loss. The result highlights the fact that if $f_{C_A} > f_{P_A}$, the steady-state will be given by the population of Type A firms playing C (*AllC*) against a population of Type B firms that withdraw from the cartel agreement (*AllD*). Therefore, opportunistic behavior, such as the free-rider, can also configure a balance that reinforces the weakness of the stability of cartel agreements.

Figure 3 – Type A and Type B Firms are All C and All D, respectively.

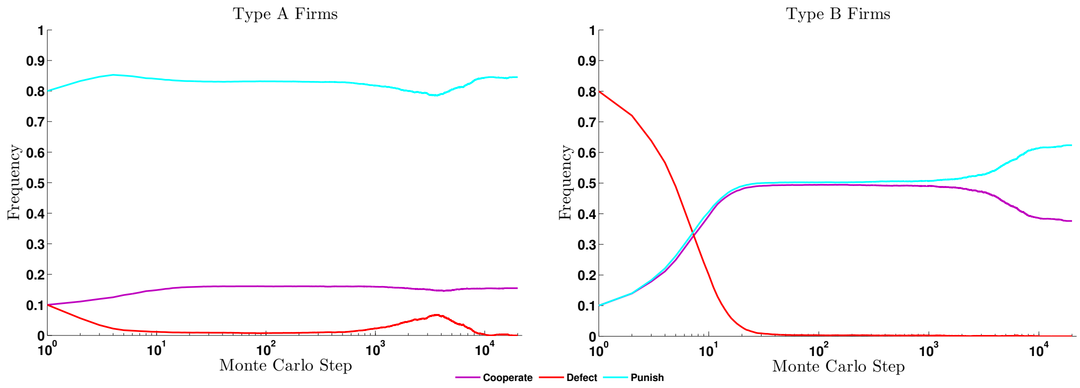


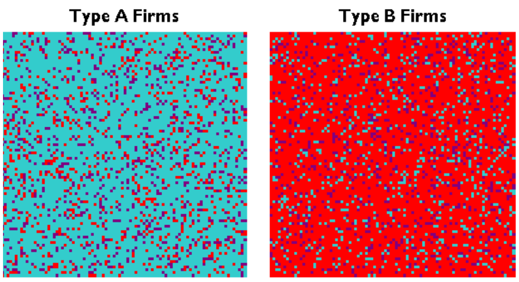
Source: Elaborated by the authors.

Result 1 summarizes and presents the economic intuition of these results.

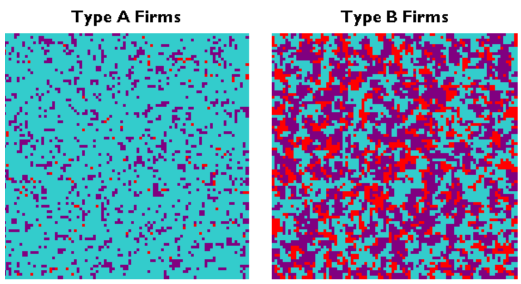
Result 1. In the Well-mixed competition, when $(f_{C_A}, f_{D_A}, f_{P_A}) = (0.1; 0.1; 0.8)$ and $(f_{C_B}, f_{D_B}, f_{P_B}) = (0.1; 0.8; 0.1)$, the punishment is effective in eliminating the deviant behavior of the market if, and only if, $\rho_A = 0.01$ and $0.01 < \rho_B \lesssim 0.27$. Another necessary condition for the punish character to be evolutionarily stable is that the frequency of punishing firms must be permanently greater than the frequency of cooperators in population A, that is, $f_{P_A} > f_{C_A}$. Otherwise, there is room for free-rider behavior and the equilibrium of the game is given by the state All C vs All D since firms of population B will have an incentive to deviate from the agreement in the absence of punitive firms in population A. For any other values assigned to ρ_A and ρ_B, the balance of the game is formed by Type A firms cooperating and Type B firms defecting at the expense of the cooperative behavior from the rival population.

Figure 4 – Steady state for the spatial competition with $(\rho_A, \rho_B) = (0.01; 0.015)$.

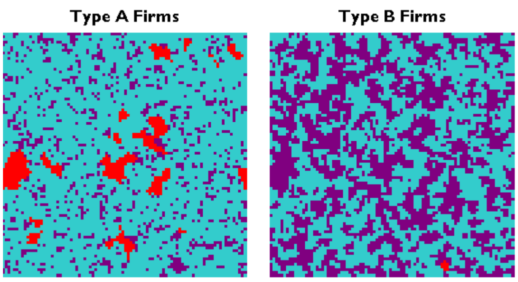




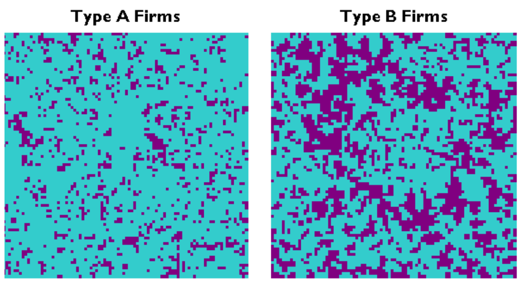
(a) MCS 1



(b) MCS 10



(c) MCS 4000



(d) MCS 12000

Source: Elaborated by the authors.

2.3.5 Spatial-structured competition

Now, we evaluate the steady states of the competition among firms restricted to their local neighborhood. Thus, preserving the previous initial conditions, we now evaluate how spatial interaction affects firms' strategic behavior. When $(\rho_A, \rho_B) = (0.01; 0.015)$, we observe two different steady states, in the same way as before. The significant difference here is that for spatial competition, punishment is much more sensitive to the variation of the cost for the benefit of mutual cooperation. In addition, it will only be effective when defectors are scattered, in the sense that they do not have a compact clustering, and when the competitive neighborhood of these firms is composed mostly by those firms who apply the punishment.

In order to illustrate our argument, we compare the outcomes presented in Figures 4 and 5. Between $0 < MCS \lesssim 10$, in both cases, we observe a decreasing in the number of defector firms in population A and B. This decrease in population B is more pronounced than in population A. In addition, simultaneously, there was a significant increase in C and P strategies in the population of Type B firms. Note that, between steps $10 \lesssim MCS \lesssim 1000$ the relative frequency of defecting firms in both populations remains quite low.

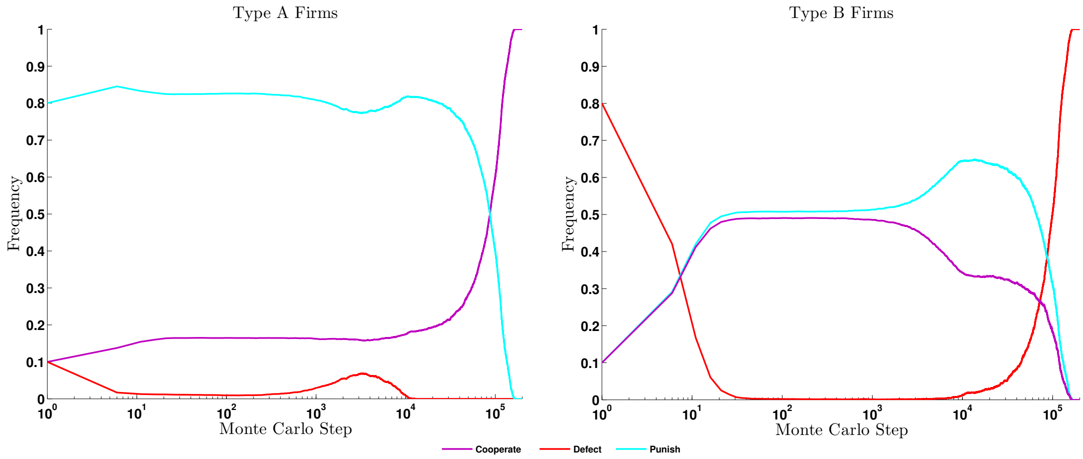
The frequency of defectors in the population of Type A firm shows a timid growth between $10 \lesssim MCS \lesssim 1000$, as shown in figure 4c and 5b. This is because strategy D is a better response to neighboring (and adversary) firms that play C. However, the growth of deserting firms is interrupted by the punishment they suffer by competing with firms that are willing to give up part of their profit to sustain the collusive agreement. Thus, simultaneously, the defectors observe that the cooperation between the other firms generates greater profit and, from then on, they update their strategy for C or P, starting to cooperate with the cartelization of the market. The net effect of this dynamic is that the relative frequency of firms playing D in both populations is very close to zero.

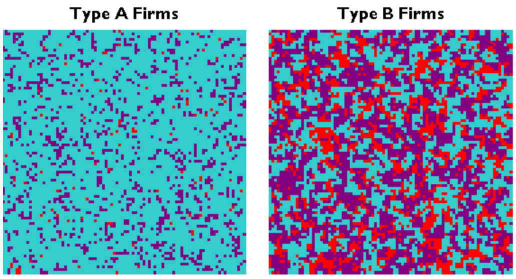
The difference between the steady states shown in Figures 4 and 5 begins to be drawn around the $MCS \gtrsim 10.000$. At this point, we can compare Figures 4d and 5d. Note that in 4d, in both populations there is a higher relative frequency of punishers. This inhibits the formation of compact clusters of defecting firms.

On the other hand, as can be seen in figure 5d, the relative frequency of punishers in the population of Type A firm decreases and the frequency of cooperators increases, becoming greater than the frequency of punishers. This situation favors deserting firms and the frequency of Type B firms defecting increases substantially. Note that there is a very compact cluster of cooperators and defectors in the population of firms of Type A and B, respectively. This happens because strategy C performs better than P when competing against deserting firms.

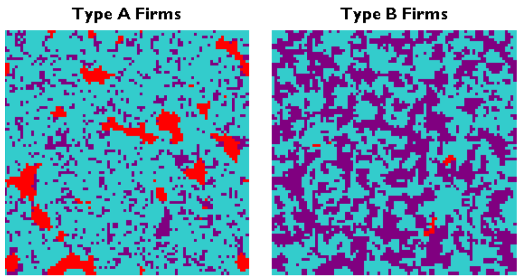
Simultaneously, the frequency of Type B firms playing D increases at the expense of the absence of punishment in the competitive neighborhood. Thus, as these clusters become more solid, the outcome of the game is formed by Type A firms cooperating and Type B firms defecting from the cartel agreement. We can summarize these outcomes in Result 2.

Figure 5 – Steady state for the spatial competition with $(\rho_A, \rho_B) = (0.01; 0.015)$.

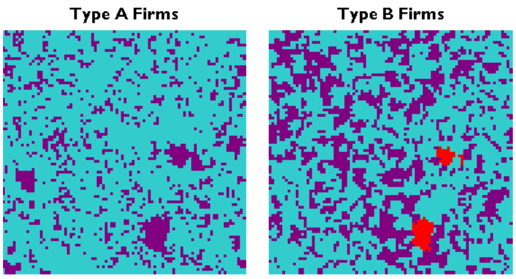




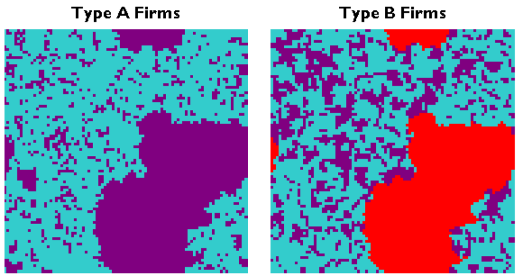
(a) MCS 1



(b) MCS 3000



(c) MCS 20000



(d) MCS 65000

Source: Elaborated by the authors.

Result 2. In an environment in which we define a relevant market with heterogeneous firms and the competition is restricted to a local neighborhood, the stability of the collusive agreement is much more sensitive to the parameters ρ_A and ρ_B. This outcome indicates that punishment is less effective in such environments. In this sense, departing from $(f_{C_A}, f_{D_A}, f_{P_A}) = (0.1; 0.1; 0.8)$ and $(f_{C_B}, f_{D_B}, f_{P_B}) = (0.1; 0.8; 0.1)$, punishment eliminates the deviant behavior of the market if, and only if, $\rho_A = 0.01$ and $0.01 < \rho_B \leq 0.015$. Another necessary condition for the punish character to be evolutionarily stable is that $f_{P_A} > f_{C_A}$ and the defecting firms from population B must be scattered in the grid.

Otherwise, there is room for free-riders and firms that adopt this behavior form a compact red cluster, as shown in Figure 5d. Therefore, the equilibrium of the game is given by the state All C vs All D, since firms of population B will have an incentive to deviate from the agreement in the absence of punitive firms in their local neighborhood. For any other values assigned to ρ_A and ρ_B, the balance of the game is formed by Type A firms cooperating and Type B firms defecting from the agreement at the expense of the collusive behavior from the rival population.

2.4 CONCLUSION

The cartel agreement was studied from an evolutionary game perspective. By doing so, we aimed to provide a better understating and evaluation of the stability of firms' collusive behavior. Supported by the bounded rationality assumption, we suggested a different way of describing the use of the combination of reward and punishment to induce and sustain a cartel agreement. Firstly, contrary to the traditional literature on cartels, we dissociate the strategy of not cooperating from strategies related to betrayal and punishment. Thus, we began our analysis on the stability of the cartel by introducing the parameter ρ, which measures the cost to the benefit ratio when both firms are joining the cartel agreement. In this way, we observed that the greater the value of ρ the greater the incentive to deviate from the cartel settlement and there still a paradox since the average profit of the industry is lower than it would be if only collusive firms existed in the steady-state.

To better capture the heterogeneity among firms willing to cooperate with the cartel agreement, we proposed a two-population game. Thus, we considered that firms had different behavior towards the cost-benefit of cooperation. We labeled as Type A those firms more efficient in the sense that they have lower costs of production, of monitoring and of organizing the cartel agreement. This turned possible for them to require a lower ρ than Type B firms, once the latter was assumed to be less efficient. In sequence, we introduced an enforcement mechanism by a punishment strategy through an Agent-Based Simulation model in our two-population game.

The punisher acts like an altruistic cooperator in the sense that it is willing to incur an extra cost to punish the defectors from the competitive environment. This has enabled us to provide more adequately answers about the nature of the stability of the cartel arrangements and how often punishment is capable of sustaining collusion. On this matter, we observed that to sustain cooperation among firms, if we set $\rho_A = 0.01$ and $0.01 < \rho_B \lesssim 0.27$, the frequency of punishment in the well-mixed two-population game has to be permanently greater than the fraction of cooperators in the population of Type A firms. Otherwise, the equilibrium of the game is formed by the population of Type A firms cooperating and the population of Type B firms defecting from the agreement.

This same result is observed for spatial competition, but punishment is only effective in eliminating the defecting firms in the range that $\rho_A = 0.01$ and $0.01 < \rho_B \lesssim 0.015$.

For future research on this topic, given the weakness of the stability of the cartel arrangements, it would be interesting to propose a game-theoretic model to evaluate the interaction between the antitrust authority and the cartelized firms. In this sense, a wide and unprecedented discussion on efficient mechanisms to inhibit cartel agreements can be established based on the bounded rationality assumption together with the agent-based simulation models.

3 ILLEGAL CARTEL

ABSTRACT

This paper offers a theoretical model for the analysis of illegal cartels. Given the nature of the cartel, retaliation is also illegal. To assess the stability of collusion as a criminal organization, we propose a one-shot game based on Bertrand competition with product differentiation. We confirm our conjectures on both the cartel's internal and external stability through numerical solutions. Depending on market parameters, the cartel remains stable with up to six homogeneous firms. By introducing cost asymmetry that number is significantly higher, and the collusion proves to be increasing in the share of high-cost firms and decreasing in the share of low-cost firms in the market.

keywords: Illegal cartels. deviation and retaliation. cartel stability.
JEL classification: C72 · D21 · D43 · K21 · L13

3.1 INTRODUCTION

The Theory of Industrial Organization and the analysis of Law & Economics emphasizes that the degree of market concentration, the number of firms operating in the same industry, as well as the asymmetry concerning efficiency and productive capacity, play an important role in the cartel's power to manipulate market prices (HOLMSTROM; TIROLE, 1989; LEVENSTEIN; SUSLOW, 2011). Although there is a growing interest in this subject − mainly guided by antitrust authorities to ensure greater effectiveness in detecting and punishing cartel members − the empirical evidence that supports this theoretical argument is still limited, based primarily on legal cartels (GRIFFIN, 1989).

Cartels are considered to be legal if they operated before the enactment of antitrust laws in the jurisdictions in which they functioned, or extra-legal if they were not known to have been punished by an antitrust authority. Other legal cartels were organized and registered under antitrust exemptions, such as export cartels or ocean shipping conferences (CONNOR, 2007; COMMISSION et al., 2007). The largely known legal cartel is the one formed by OPEC, which is organized by sovereign states. Under traditional legal views, it cannot be held to antitrust enforcement in other jurisdictions under the doctrine of state immunity under public international law (FARAH; CIMA, 2013).

However, the purpose of this paper is to study illegal cartels. Narcotics and gambling are traditional examples. Empirical evidence suggests that cartels acting illegally manage to attain the same or even higher levels of overcharges as legal cartels. This may imply that illegal cartel agreements become more sophisticated and cartel participants manage to enforce them very effectively taking both the economic and legal environment into account. This argument has been reinforced by hardcore cartels[1] (BOLOTOVA; CONNOR; MILLER, 2008). As well, the illegal aspect of firms' market behavior turns the intersection between Law & Economics and Industrial Organization approaches even more relevant.

Therewith, we aim at proposing a discussion on the theoretical motivations that guide firms' engagement in illegal activities. In this regard, we embrace the cost to benefit analysis as in the Theory of Collusion (BECKER, 1968). Typically, the benefits of competing firms to collude is related to the monopoly profits, which increases in the elasticity of firms' marginal cost curves and decreases in the elasticity of firms' collective demand curve. When a firm violates the collusion whether pricing below or producing more than is agreed, this opportunistic behavior is harmful and offensive to the collusion. In that sense, there are costs as well as different strategies for eliminating violations. The first costs stem from the effort to discover and apprehend the defector. That done, there is a cost to the collusion in punishing these defectors. This approach has proved to be quite

[1] For details, see Cartels (2003), Hüschelrath and Weigand (2013), Clemens and Rau (2019).

useful in understanding how agreements involving unlawful activities, such as mafia-type organizations, whether as criminal enterprises, or as illicit networks, or a special form of organized crime such as illegal cartels, establish the set of rules and punishment strategies to ensure stability (BLANCKENBURG; GEIST, 2011; SERGI, 2019).

In microeconomic theory, the legal cartels' stability is associated with market characteristics that would or would not favor collusive behavior. Another relevant issue in the analysis is to measure how profitable the violation would be for the violator (TREMBLAY; SCHROEDER; TREMBLAY, 2018). In the context of repeated games, based on the sub-perfect Nash equilibrium (SPNE) concept, the discount factor must be sufficiently close to 1 for a grim trigger strategy[2] to support cartel stability. In other words, in terms of the expected payoff, future benefits are equivalent to the current benefits (FRIEDMAN, 1971). Otherwise, any deviation from the cartel output restriction is met with permanent reversion to one-shot-game non-cooperative equilibrium values.

Still, there is a wide discussion about the optimal number of firms participating in a cartel. Many approaches consider the existence of a competitive fringe (Bertrand or Cournot), based on simple demand and cost functions to derive the size of the stable cartel (SHAFFER, 1995). In some collusive markets with competitive fringe, it is still possible to observe the spillover effect from the cartel activity, i.e., firms capture the highest influence from the cartel without actually participating in it. This situation could change the cartels' stability (KAMIEN; MULLER; ZANG, 1992). A cartel is stable if firms inside the cartel do not find it desirable to exit and firms outside the cartel do not find it desirable to enter (D'ASPREMONT et al., 1983). Since the 80s the intuition behind coalition structures discussed in Game Theory has been considered in the oligopoly context. Typically, these models are based on comparative-static analysis and show that the way competition takes place determines whether collusion is more or less attractive (DONSIMONI; ECONOMIDES; POLEMARCHAKIS, 1986; THORON, 1998). Under this motivation, many studies aim to prove the existence and discuss the size of stable cartels with a range of different demand and cost functions through both analytical and numerical approaches (ZU; ZHANG; WANG, 2012; PAPAHRISTODOULOU, 2019).

Given the above, our paper is innovative in its theoretical approach in examining the stability of an illegal cartel. We propose a one-shot game with retaliation. Besides, motivated by the punishment strategies applied by criminal organizations, the way retaliation takes place is also illegal. The intuition of this approach is as follows. Since collusion is typically prohibited by law, illegal methods of retaliation can be more harmful to defectors, as they do not have legal remedies to fight back. To achieve that goal, we asses a regional market with a Bertrand fringe in both symmetric and asymmetric oligopoly. Due to the algebra complexity, we made use of the numerical analysis as a robustness check to

[2] See Feuerstein (2005) for a survey.

complement the discussion of our findings (KONISHI; LIN, 1999; OLIEMAN; HENDRIX, 2006).

In summary, the main results of this paper are as follows: i) For the case of homogeneous firms, we show that the cartel's stability is guaranteed for an industry with a maximum of six high-efficiency firms[3] and an extremely low degree of product differentiation; ii) Within an industry with heterogeneous firms, cartel's stability is found to be increasing (decreasing) in the number of high (low)-cost firms in the regional industry as well as to be decreasing in cost asymmetries; iii) Conditional on the type of firm that intends to deviate from the cartel agreement, our findings suggest the existence of different settings for the stable cartel, making it possible to identify the necessary conditions for the collusion to hold.

The remainder of this paper is organized as follows. In Section 3.2 we review the existing literature. Section 3.3 introduces the model and derive both the external and internal stability of the illegal cartel in many different settlements. Section 3.4 concludes.

3.2 LITERATURE REVIEW

Since our approach is about illegal cartels, this paper is in line with the strand of Law & Economics literature on organized crime inspired by Becker (1968). In his seminal work, Becker uses economic analysis to model illegal behavior. More precisely, he thought of crime in rational terms, arguing that potential criminals would trade off the gains from crime against the expected costs. Since then, the theoretical literature has outspread the economic view of deterrence in many different ways. Ehrlich (1973) discusses the engagement in illegal activities as a time-allocation problem. There is also a strand of literature developed from the increment of the Becker model. Kaplow (1990), Bebchuk and Kaplow (1992), Levitt (1997), Garoupa (1999) uses limited information. Polinsky and Rubinfeld (1991) includes repeat offending. There is also a debate about enforcement errors and the corruption of law enforcers, as in Png (1986), Bowles and Garoupa (1997), Polinsky and Shavell (2001), Silva, Kahn and Zhu (2007).

Even closer to our approach, Gambetta and Reuter (1995), Blanckenburg and Geist (2011) discuss the interest as well as the criminal strategies of the mafia as cartel enforcers. This led directly to a theory of deterrence intending to predict patterns of criminal behavior. Following this framework, Khadjavi (2018) explains crime and assess punishment[4] in a controlled environment with complete information. Assuming that risk-neutral individual engages in crime, the author shows that if the expected benefit is

[3] Whose production cost $k \to 0$.
[4] For a survey on the empirical study of criminal punishment see Levitt and Miles (2007).

less than the expected fine[5], deterrence incentives work to reduce the stealing of criminals and, consequently, decreases the stability of the unlawful coalition.

On the Theory of Industrial Organization[6], we mention studies derived from the formation of coalitions in oligopolies that are modeled as non-cooperative games in which firms' strategies are to cooperate or to cheat the cartel agreement. The seminal contribution uses comparative-static analysis and is given by Salant, Switzer and Reynolds (1983), who evaluated how an exogenous change in industry structure motivated by collusion affects the Nash equilibria in a Cournot model. A well-known result about legal cartels' stability shows that cooperation is not feasible in the one-shot game due to the incentive of firms to deviate unilaterally from the agreement. On the other hand, this result can be reversed in the repeated game - in which one subgame perfect Nash equilibrium (SPNE) supports cooperation Tirole (1988).

In a repeated-game model of collusion, the stability is inversely related to the discount factor. Studying a dynamic noncooperative model of collusion with demand uncertainty, Green and Porter (1984) concludes that collusion essentially ends after some rounds of interaction. As imperfect information makes it impossible for firms to know that other firms are cooperating, the punishment works as a permanent reversion mechanism to competitive pricing. Taking this into account, Levenstein and Suslow (2011) evaluates the determinants of cartel duration and analytically demonstrates how an unanticipated increase in the market interest rate may destabilize a cartel. Barsky and Kilian (2004) empirically discuss the impact of fluctuations in the interest rate on the stability of the OPEC cartel[7].

Among the various developments on cartel stability, the theoretical contribution is given by d'Aspremont et al. (1983) on external and internal stability stands out. On this subject, Donsimoni, Economides and Polemarchakis (1986) shows that for specifics values of the cost parameter, two stable cartels exist. Since then, there have been many refinements to this approach. Here, we emphasize the concept of "coalition-proof", which takes into account not only the diversion of a single but of several firms - which may come to be grouped into sub-coalitions. This analysis proves to be somewhat more rigorous, given the need to calculate the stability conditions of each of the possible deviation paths. Deneckere and Davidson (1985) used a similar approach in a Bertrand model to assess

[5] The expected fine is given by (probability of detection) × (punishment cost). See Silva, Kahn and Zhu (2007) for details.

[6] As well, the game-theoretical concepts presented here are of fundamental importance in a wide variety of literature, such as international environmental agreements Ecchia and Mariotti (1998), Diamantoudi and Sartzetakis (2006), Silva, Zhu et al. (2015), Ansink, Weikard and Withagen (2019), Finus and McGinty (2019), local public goods provision and political interaction Cross (1967), Greenberg and Weber (1993), Montero (2006), Sun, Trockel and Yang (2008) and transnational terrorism Oliveira, Faria and Silva (2018).

[7] Jr (1989) provides a detailed discussion of collusion with asymmetric discount factors.

the endogenous formation of coalition structures. Results show that the incentives to cooperate are more prone when strategic actions are complements rather than substitutes. Moreover, Bernheim, Peleg and Whinston (1987) outlined a Bertrand game model to demonstrate how the coalition-proof is useful in situations where firms can engage in pre-communication but cannot establish biding contracts[8]. Inspired by these authors, Thoron (1998) define the concept of the Coalition-Proof Nash Equilibrium (CPNE)[9] to prove that the set of stable cartels is unique. Contrasting the results on the uniqueness of the set that characterizes the stable cartel, we can mention Zu, Zhang and Wang (2012). Prokop (1999) represents the process of collusion through extensive form games in which each firm decides to cooperate with the cartel or not. Applying subgame perfect equilibrium it yields the same results regarding stable cartel sizes found by d'Aspremont et al. (1983), Diamantoudi (2005).

Following Selten (1973), all these studies assume that the cartel behaves as a leader to the competitive fringe. Another common feature is the existence of some enforcement mechanism for collusion such that once a firm decides to join the cartel, there is no possible cheating on the agreement. Withal, binding collusion is known to be an illegal agreement. This fact led to the emergence of a strand in the literature related to tacit collusions (FRIEDMAN, 1971; MARTIN, 1993; PROKOP, 1999). Focusing on firms' incentives, this approach aims to analyze the symmetric (SPNE) that maximizes industry profits. Escrihuela-Villar (2009) demonstrates how the sequence of actions between the cartel and the fringe affects the tacit collusion in a Cournot competition[10].

Finally, our theoretical approach stands out from the existing literature in the following aspects: (i) The cartel is illegal; (ii) The one-shot game has two stages, within the first one firms are deciding whether to join the cartel or not. In the second period, faced with the possibility of a firm betraying the agreement, there is retaliation by the criminal organization;(iii) Due to costs asymmetries, firms in the cartel adopt different prices - depending on their type (high or low-cost). The main motivation for this price behavior is twofold: (a) it makes inspection more difficult for the regulatory agency - which reduces the likelihood of the cartel being discovered and, consequently, decreases the expected value of the fine; (b) it is a mechanism to reward the most efficient firms that adhere to the agreement.

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8 Routledge (2013) shows that in such situations any deviation must be self-enforcing.

9 Thoron (1998) defines a CPNE as a strategy profile that is robust to self-enforcing deviations.

10 See Bloch (1996), Currarini and Marini (2015) for a survey of the literature on stable horizontal mergers in Cournot games. Lardon (2019) revisit both the Bertrand and Cournot oligopolies and discusses the coalitional stability of the game in the presence of a cartel within a competitive fringe. Dugar and Mitra (2016) evaluated the cartel stability in a Bertrand competition with asymmetric marginal costs. Papahristodoulou (2019) proposed a model in which the cartel is the Stackelberg leader and the followers are competing in a Cournot Fringe. Results suggest that the number of firms in the cartel is lower than the one found by d'Aspremont et al. (1983).

3.3 MODEL

Consider an economy with r regions. In each region $i, i = 1...r$, there is an oligopolistic industry consisting of $n_i \geq 3$ firms. Each regional industry sells differentiated products. There are two periods. In each period, if firm j in region i, chooses a price $p_{j,i}$ for its product, the quantity demanded of this firm's product is:

$$q_{j,i}(p_{j,i}, \bar{p}_i) = b_i - p_{j,i} - \delta_i(p_{j,i} - \bar{p}_i), \tag{3.1}$$

where b_i represents the highest price that a consumer from a given region i is willing to pay, i.e., the market reservation price (MRP). Parameter $\delta_i \in (0, 1]$ captures the degree of product differentiation in each region. In other words, when it is equal to zero (one), the goods are independent (homogeneous). We denote by $\bar{p}_i = n^{-1} \sum_{k=1}^{n} p_{k,i}$ the industry's average price. For simplicity, we first assume that firms are symmetric, i.e., the constant per unit cost of supplying any product is k.

3.3.1 The antitrust authority

An antitrust agency is in charge of preventing price fixing (cartelization) among firms in the entire economy. This regulator, however, has a fixed budget T, set by the economy's government. The larger the budget, the greater the number of inspections. Let $D_i = \{1, ..., d_i\}$ and $H_i = \{d_i + 1, ..., h_i\}$ denote the sets of dishonest (law-breaking) and honest (law-abiding) firms in region i, where $d_i \geq 0$ and $h_i \geq 0$ are the numbers of dishonest and honest firm in the region, respectively. Note that $d_i + h_i = n_i$. As we consider deviations during the action game from firms that agree to join the cartel during pregame communications, it is important that we define the set of active cartel members. This is set $M = \{1, ..., m_i\}$, where $m_i \leq d_i$.

Let σ_i denote the probability of conviction faced by each dishonest firm in region i. We consider that the number of firms in each region may vary and is given by the vector $R(G) = \{n_1(g_1), ..., n_i(g_i)\}$. As we consider that regions are asymmetric in relation to the concentration of firms, the vector G captures the geographical aspects as well as the local concentration of firms in the same region. Thus, the probability of conviction is a function of $R(G)$ and T: $\sigma_i = \sigma_i(R, T)$. We assume that σ_i increases with T and decreases with R.

Hence, as the regulator has a fixed budget, the greater the concentration of firms of the same industry in a given geographic region: (a) the greater the willingness to price collusion; (b) the lower the probability of a dishonest firm being caught; (c) the greater the regulator's efforts to prevent the cartel. If the dishonest firms in region i are convicted, each firm must pay a fine $\mathcal{F} = f$ to the government.

Now, we offer an intuition about the role of the regulatory agency. First, assume that the antitrust authority can only rely on inspections to detect collusion. Regarding evidence of collusion, we assume that collusion cannot occur without communication among the firms and that communication generates hard evidence, such as memos and reports of meetings (BELLEFLAMME; PEITZ, 2015). The profits firms can obtain in that case are as follows. In the polar case where no firm engage in communications in region i, each firm earn the oligopoly profit π_i^O. If firms communicate, they earn the collusive profit discounted by the expected value of the fine, given by $\pi_i^M - \sigma_i f$. If one firm deviates, it increases its profit to $\pi_i^D - \sigma_i f$. Typically, the regulator consider that collusion can be sustained if:

$$\frac{\pi_i^M - \sigma_i f}{1-\lambda} \geq \pi_i^D - \sigma_i f + \frac{\lambda \pi_i^O}{1-\lambda},$$

which yields to

$$\frac{\lambda}{1-\lambda}(\pi_i^M - \pi_i^O - \sigma_i f) \geq \pi_i^D - \pi_i^M,$$

where $\lambda \in (0,1]$ is the discounted factor. In this way, the antitrust authority may bring instability to cartel agreements as they increase the expected value of the loss from punishment (left-hand side), by increasing $\sigma_i f$. This would cause the collusion's profit to become less than the immediate gain from the deviation (right-hand side). Thus, the above condition does not hold. In the following subsection, we assess the collusion from the firms' perspective.

3.3.2 The setting with 3 homogeneous firms

We start the analysis by examining a setting in which $n_i = 3$. As firms are symmetric, their costs are given by $k \in [0,1]$ with $b_i > max\{0,k\}$. We first consider the polar case in which all firms are honest. This provides us with a useful benchmark for future comparisons. If the three firms are honest, in each period, firm j chooses non-negative $p_{j,i}$ to maximize the oligopoly profit as follows:

$$\pi_{j,i}^O = \frac{1}{3}(p_{j,i} - k)[3b_i + \delta_i P_{-j,i} - p_{j,i}(3 + 2\delta_i)], \tag{3.2}$$

where $P_{-j,i} = P_i - p_{j,i}$ and $P_i = \sum_{k=1}^{3} p_{k,i}$, taking the other firm's price choices as given. The first order conditions (F.O.C.) is detailed in A and yield the following payoff in each period:

$$\pi_{j,i}^O = \pi_i^O = \frac{(b_i - k)^2(3 + 2\delta_i)}{4(3 + \delta_i)^2} \qquad j = 1, 2, 3. \tag{3.3}$$

Consider now the other polar case, where all firms are dishonest and belong to the cartel. Hence, $M = D = \{1, 2, 3\}$. In each period, the cartel chooses non-negative $\{p_{1,i}, p_{2,i}, p_{3,i}\}$ to maximize the following expected payoff, taking σ_i and f as given:

$$\sum_{F=1}^{3} E\pi_{F,i}^M = \sum_{F=1}^{3} \left\{ \frac{1}{3}(p_{F,i} - k)[3b_i + \delta_i P_{-F,i} - p_{F,i}(3 + 2\delta_i)] - \sigma_i f \right\}. \tag{3.4}$$

The F.O.C is derived in A and imply that each firm earn the following expected payoff in each period:

$$E\pi_{j,i} = E\pi_i^M = \frac{b_i(b_i - 2k)}{4} - \sigma_i f. \tag{3.5}$$

3.3.3 Deviation and retaliation

We now consider two situations where the cartel contains two firms, say, firms 1 and 2. We first examine a situation where all firms agree to join the cartel prior to the beginning of the game, but firm 3 deviates in the first period. The cartel observes the deviation and retaliates in the second period. In the retaliation period, the cartel moves first. Retaliation takes the form of stealing some (or all) of the defector's product. Later, we examine a situation where firm 3 decides not to join the cartel prior to the beginning of the game. In this case, the cartel retaliates in each period.

Suppose that during pre-game communications, all firms agree to join the cartel. During the action game, however, firm 3 deviates. Hence, $D = \{1, 2, 3\}$ and $M = \{1, 2\}$. Consider the first period. Firms 1 and 2 set the cartel price derived in A. By it turn, firm 3 takes this price into account and then chooses non-negative $p_{3,i}$ to maximize:

$$\pi_{3,i} = (p_{3,i} - k)q_{3,i} = \frac{1}{3}(p_{3,i} - k)[3b_i + (b_i + 2k)\delta_i - p_{3,i}(3 + 2\delta_i)] \tag{3.6}$$

Hence, firm 3's expected payoff in the first period is

$$\pi_{3,i} = \frac{[(3 + \delta_i)b_i - 3k]^2}{12(3 + 2\delta_i)} - \sigma_i f. \tag{3.7}$$

Each cartel member earns the following expected payoff in the firs period:

$$E\pi_i^M = \frac{[9 + \delta_i(6 - \delta_i)]b_i^2 - 3(6 + 5\delta_i)b_i k}{12(3 + 2\delta_i)} - \sigma_i f \tag{3.8}$$

In the second period, the cartel moves first and retaliates. The cartel steals a quantity s from the defector and sets its prices knowing how the defector will react. We assume that the cartel faces a cost c per unit of quantity stolen from the defector, with $c \in (0, b_i)$. The defector observes $\{p_{1,i}, p_{2,i}, s\}$ and chooses non-negative $p_{3,i}$ to maximize

$$\pi_{3,i} = \frac{1}{3}(p_{3,i} - k)[3b_i + \delta_i P_{-3,i} - p_{3,i}(3 + 2\delta_i) - s]. \tag{3.9}$$

The cartel anticipates how the defectors will react. It chooses non-negative $\{p_{1,i}, p_{2,i}, s\}$ to maximize

$$\begin{aligned} &\frac{1}{3}\Bigg\{(p_{1,i} - k)[3b_i + \delta_i P_{-1,i} + \frac{s}{2} - p_{1,i}(3 + 2\delta_i)] + (p_{2,i} - k)[3b_i + \delta_i P_{-2,i} + \\ &\frac{s}{2} - p_{2,i}(3 + 2\delta_i)]\Bigg\} - cs \\ &\text{s.t.} \quad s \leq 3b_i + \delta_i P_{-3,i} - (3 + 2\delta_i)k \end{aligned} \tag{3.10}$$

The constraint follows from firm 3's quantities derived in equation (3.9) and the fact that the quantity sold by firm 3 cannot be negative. Since the objective function (3.10) is linear in s, we obtain

$$s = 3b_i + \delta_i P_{-3,i} - (3 + 2\delta_i)k \quad if \quad p_{1,i} + p_{2,i} \geq 6c, \tag{3.11}$$

$$s = 0 \quad if \quad p_{1,i} + p_{2,i} < 6c. \tag{3.12}$$

Assume initially that the inequality in (3.11) holds. Then, the cartel chooses non-negative $\{p_{1,i}, p_{2,i}\}$ to maximize

$$\begin{aligned} &\frac{1}{2}\left\{(p_{1,i} - k)[3b_i + \delta_i p_{2,i} - p_{1,i}(2 + \delta_i)] + (p_{2,i} - k)[3b_i + \delta_i p1, i + -p_{2,i}(2 + \delta_i)]\right\} \\ &- c[3b_i + \delta_i(p_{1,i} + p_{2,i}) - (3 + 2\delta_i)k]. \end{aligned} \tag{3.13}$$

From equation (3.13) we have that

$$s = \frac{3(2 + \delta_i)}{2} b_i - \delta_i^2 c - (3 + \delta_i)k, \tag{3.14}$$

$$c \leq \frac{3}{2(6 + \delta_i)} b_i + \frac{k}{(6 + \delta_i)}. \tag{3.15}$$

Inequality (3.15) is the necessary condition for the cartel to steal firm 3's product. Furthermore, each cartel member's expected payoff is given by

$$E\pi_i^M = \frac{9b_i^2 - 4\delta_i^2 c^2}{8} + \frac{k - (3b_i - 2\delta_i)}{2} k - \sigma_i f. \tag{3.16}$$

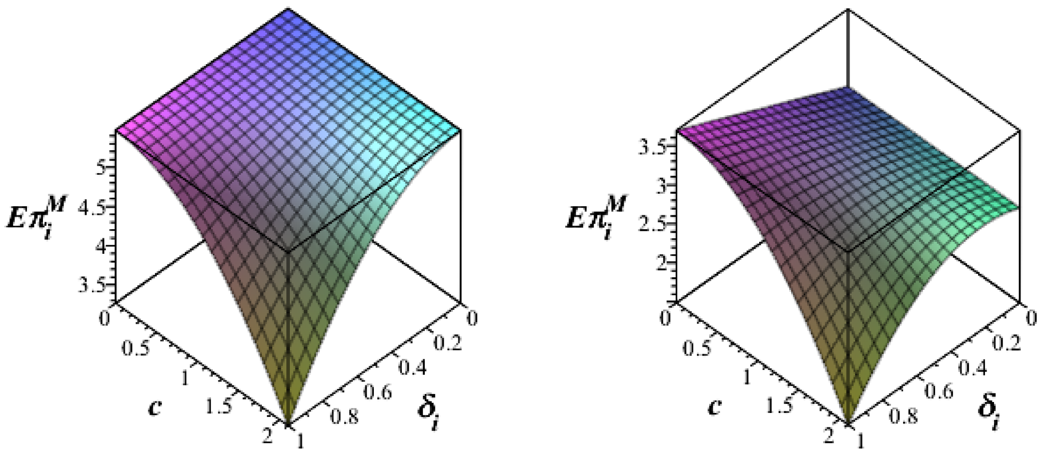
The expected payoff earned by firm 3 is

$$E\pi_{3,i} = -\sigma_i f. \tag{3.17}$$

By definition, when the inequality in (3.15) holds, the cartel is stable[11]. However, we offer a brief comparative statics showing how the cost of stealing c affects each cartel member's expected payoff as given in (3.16). It is straightforward to see that the lower c the greater $E\pi_i^M$. Figure 6a shows the stability when condition 3.15 holds and $k \to 0$. Figure 6b illustrates how the stability holds when $k \to 1$. As s is linear in the objective function (3.10), this relationship between the cost of stealing and the cartel stability does not change even if there are N homogeneous firms in the market - as we evaluate in Subsection 3.3.4.

[11] As the payoff in (3.7) is always greater than (3.16), the external stability always holds. In the same way, the payoff in (3.16) is always greater than the payoff in (3.17) - which guarantees the internal stability.

Figure 6 – Cartel stability according to c and δ_i.



(a) $b_i = 2.21; k \to 0$ (b) $b_i = 2.21; k \to 1$

Source: Elaborated by the authors.

Suppose now that the inequality in (3.12) holds. Then, the cartel chooses non-negative $\{p_{1,i}, p_{2,i}\}$ to maximize

$$\frac{1}{6(3+2\delta_i)} \sum_{F=1}^{2} (p_{F,i} - k)\left[3b_i + \delta_i\left(\frac{(3+\delta_i)b_i + (3+4\delta_i)k}{2(3+2\delta_i)} + p_{-F,i}\right) - p_{F,i}(3+2\delta_i)\right], \quad (3.18)$$

where $p_{-F,i} = p_{2,i}$ if $F = 1$ and $p_{-F,i} = p_{1,i}$ if $F = 2$. The expected payoff earned by each cartel member is given by:

$$E\pi_i^M = \frac{[(18+15\delta_i+\delta_i^2)(b_i-3k)][(18+15\delta_i+\delta_i^2)b_i - (18+15\delta_i-\delta_i^2)k]}{16(3+2\delta_i)^2(3+\delta_i)} - \sigma_i f, \quad (3.19)$$

Firm 3's expected payoff is

$$E\pi_{3,i} = \frac{\left\{[9(6+8\delta_i+3\delta_i^2)+\delta_i^3]b_i - [9(6+8\delta_i+3\delta_i^2)+3\delta_i^3]k\right\}^2}{48(3+2\delta_i)^4(3+\delta_i)^2} - \sigma_i f, \quad (3.20)$$

The last case to consider is the one in which during the pre-game communications firms 1 and 2 decide to form the cartel, while firm 3 decides to stay out. Hence, $D = M = \{1, 2\}$ and $H = \{3\}$. In this situation, the cartel moves first and retaliates against firm 3 in both periods. The game played in each period is identical to the game that the cartel and firm 3 play in the retaliation period in the case examined just before this one. Hence, if condition (3.15) holds, equation (3.16) is the expected payoff per period for each cartel member. The payoff per period for firm 3 is $\pi_{3,i} = 0$.

if condition (3.15) does not hold, equation (3.19) is the expected payoff per period for each cartel member. The payoff per period for firm 3 is

$$\pi_{3,i} = \frac{\left\{[9(6+8\delta_i+3\delta_i^2)+\delta_i^3]b_i - [9(6+8\delta_i+3\delta_i^2)+3\delta_i^3]k\right\}^2}{48(3+2\delta_i)^4(3+\delta_i)^2}. \quad (3.21)$$

Employing the concept of internal and external stability to characterize a stable cartel, we now show the conditions under which $M = \{1, 2, 3\}$ is stable. By definition, this set is externally stable. Hence, we only need to establish the conditions under which it is also internally stable. Assume throughout that the payoff in period 2 is valued at the same rate as the payoff in period 1; that is, the inter-temporal discount rate is zero.

If $M = \{1, 2, 3\}$ is formed during pregame communications, a unilateral deviation during the action game takes us to the setting in which one firm deviates in the first period and the other two firms, which remain in the cartel, retaliate in the second period. As before, let firm 3 be the defector. Thus, $M = \{1, 2\}$ during the action game. Assume that condition (3.15) holds. When it defects firm 3's total payoff is

$$\upsilon_{3,i} = \frac{[(3+\delta_i)b_i - 3k]^2}{12(3+2\delta_i)} - 2\sigma_i f. \tag{3.22}$$

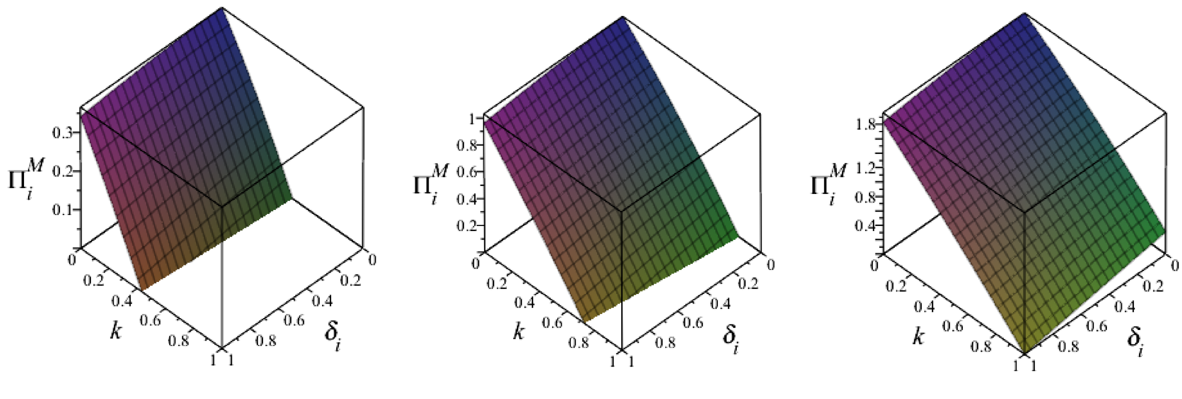
If firm 3 does not defect and thus $M = \{1, 2, 3\}$ during the action game, it earns

$$\upsilon_i^M = \frac{b_i^2}{2} - b_i k - 2\sigma_i f. \tag{3.23}$$

Comparing (3.22) and (3.23), we obtain

$$\upsilon_i^M - \upsilon_{3,i} > 0 \quad \rightarrow \quad \phi b_i^2 - \theta k > 0, \tag{3.24}$$

Figure 7 – Internal stability according to δ_i, b_i and k.



(a) $b_i = 1.21$ (b) $b_i = 2.03$ (c) $b_i = 2.80$

Source: Elaborated by the authors.

where $\phi(\delta_i) = \frac{9+\delta_i(6-\delta_i)}{12(3+2\delta_i)}$ and $\theta(\delta_i, b_i, k) = \frac{6(1+\delta_i)b_i+3k}{4(3+2\delta_i)}$. We summarize our findings for the setting with 3 symmetric firms in Result 1 as follows.

Result 1. Assuming that $k = 0$, it is easy to see that ϕ is positive because $\delta_i \leq 1$. Hence, firm 3 has no incentive to deviate from the cartel during the action game. It follows that

$M = \{1, 2, 3\}$ is stable if condition (3.15) holds. Figure 17 illustrates through a numerical example under which conditions the internal stability of the cartel is sustained. Note that as b_i increases, the larger the difference between ϕb_i^2 and θk. We can also observe that when we set a value for b_i, the value of k that determines the internal stability of the cartel is increasing in δ_i. Thus, in Figure 17a, we observe that for $\delta_i = 0$, the cartel is internally stable if $k \in [0, 0.5]$. In its turn, for $\delta_i = 1$, $k \in [0, 0.429]$. Figures 17b and 7c illustrates how an increase in the value of b_i contributes to the internal stability relative to k.

3.3.4 The setting with N homogeneous firms

With the contributions of the previous cases in mind, now consider the case with N homogeneous firms, that is, $n_i = N$. Payoffs derivations are available in A. As firms are symmetric, their costs are given by $k \in [0, 1]$ with $b_i > max\{0, k\}$. For the case where all firms are dishonest and belong to the cartel, we have $M = D = \{n_1, ..., N\}$. In each period, the cartel chooses non negative $\{p_i\}$ to maximize the expected payoff, taking σ_i and f as given:

$$\sum_{F=1}^{N} E\pi_{F,i}^{M} = \sum_{F=1}^{N} \left\{ \frac{1}{N}(p_{F,i} - k)[Nb_i + \delta_i P_{-F,i} - p_{F,i}(N + (N-1)\delta_i)] - \sigma_i f \right\}. \tag{3.25}$$

Each firm earn the following expected payoff in each period:

$$E\pi_{j,i} = E\pi_i^M = \frac{b_i(b_i - 2k)}{4} - \sigma_i f. \tag{3.26}$$

Note that the profit of the cartel does not depend on N. Consider now a situation where the cartel contains $N-1$ firms. Suppose that during the pre-game communications, all N firms agree to join the cartel. During the action game, however, firm $n_{-j,i}$ deviates. Hence, $D = \{n_{1,i}, ...N_{i,j}\}$ and $M = \{n_{1,i}...N-1_{j,i}\}$. Firms in the cartel set:

$$p_{1,i} = p_{N-1,i} = p_i^M = \frac{b_i}{2} + k \tag{3.27}$$

By it turn, firm $n_{-j,i}$ takes (3.27) into account and then chooses non-negative $p_{-j,i}$ to maximize:

$$\pi_{-j,i} = (p_{-j,i} - k)q_{-j,i} = \frac{1}{N}(p_{-j,i} - k)\Big\{Nb_i + (N-1)p_i^M\delta_i - p_{-j,i}[N + (N-1)\delta_i]\Big\}. \tag{3.28}$$

Hence, firm $n_{-j,i}$'s expected payoff in the first period is

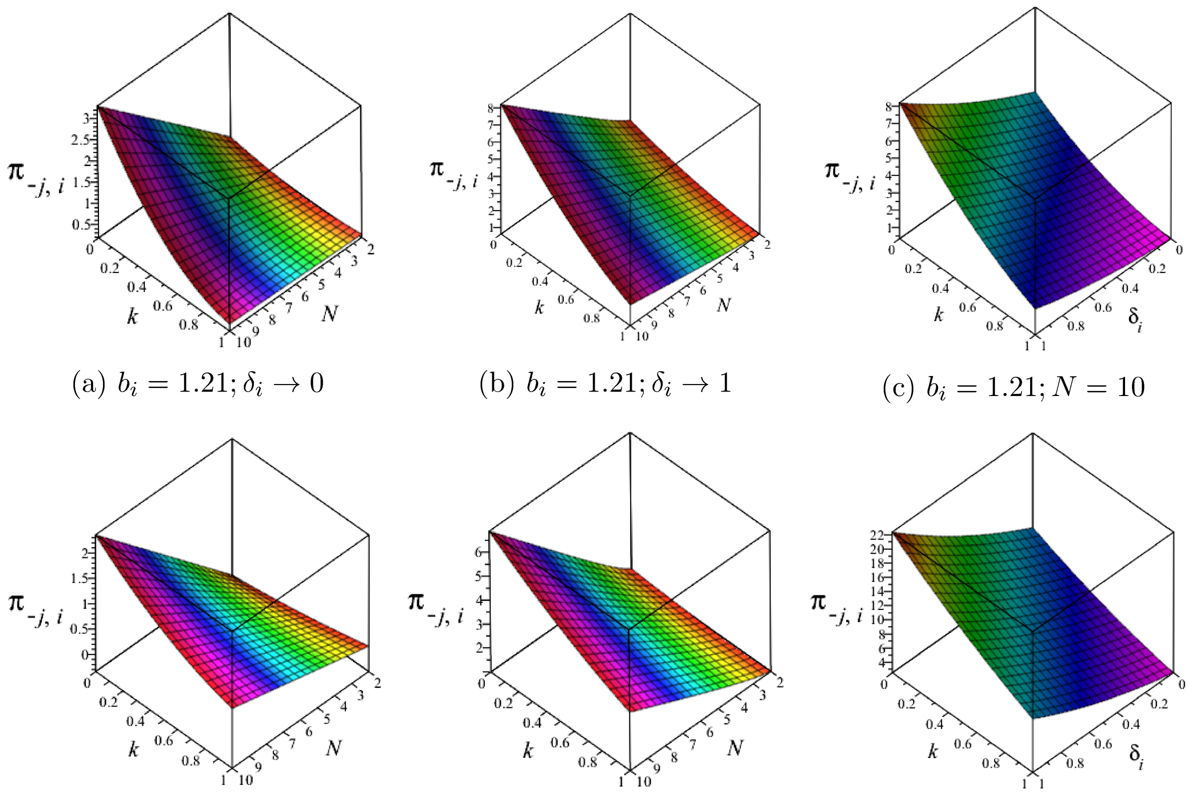
$$\pi_{-j,i} = \frac{1}{16}\frac{\Big\{[N(2+\delta_i) - \delta_i]b_i - 2Nk\Big\}^2}{3[N(1+\delta_i) - \delta_i]} - \sigma_i f. \tag{3.29}$$

Each $N-1$ cartel members earns the following expected payoff in the first period:

$$\pi_i^M = \frac{1}{8}\frac{b_i\left[2\,(1+\delta_i)\,(b_i - 2\,k)\,N^2 - \delta_i\,(b_i\delta_i + 2\,b_i - 2\,k)\,N + b_i{\delta_i}^2\right]}{N\,[N\,(1+\delta_i) - \delta_i]} - \sigma_i f. \tag{3.30}$$

Now, as before, we employ the concept of external and internal stability to characterize a stable cartel.

Figure 8 – External stability according to N, δ_i and k.



(a) $b_i = 1.21; \delta_i \to 0$
(b) $b_i = 1.21; \delta_i \to 1$
(c) $b_i = 1.21; N = 10$
(d) $b_i = 2.03; \delta_i \to 0$
(e) $b_i = 2.03; \delta_i \to 1$
(f) $b_i = 2.03; N = 10$

Source: Elaborated by the authors.

3.3.5 External stability

Typically, a cartel is said to be externally stable if it is not profitable for a fringe firm to join the cartel. Thus, from equations (3.29) and (3.30) we can make a comparative static exercise. Therefore, for the cartel to be externally stable, the following condition must hold: $\pi_i^M \leq \pi_{-j,i}$. We illustrate the evolution of external stability regarding the number of firms (N) in the market and the parameter k, which reflects the cost of production. For the sake of convenience, we normalized the condition of external stability as represented on the Z axis of Figure 8. Therefore, $\pi_{-j,i} \geq 0$ is a sufficient condition for the external stability. Through Figures 8a, 8b, 8d, and 8e it is straightforward to see that $\pi_{-j,i}$ is increasing with N and b_i. On the other hand, it is decreasing with k. Note that by keeping b_i and N constant in Figures 8c and 8f, external stability is also increasing in δ_i.

3.3.6 Internal stability

If $M = \{1, .., N\}$ is formed during pre-game communications, a unilateral deviation during the action games takes us to the setting in which one firm deviates in the first period and the other $N - 1$ firms, which remain in the cartel, retaliate in the second period.

As before, let firm $n_{-j,i}$ be the defector. Thus, $M = \{1, .., N-1\}$ during the action game. Assume that the following condition holds:

$$s = N[b_i - (1+\delta_i)k] + P_{-j,i}\delta_i + \delta_i k \quad if \quad P_{-j,i} \geq 2Nc. \tag{3.31}$$

In this case, firm $n_{-j,i}$'s total payoff is as follows when it defects

$$\upsilon_{-j,i} = \frac{1}{16}\frac{\Big\{[N(2+\delta_i) - \delta_i]b_i - 2Nk\Big\}^2}{3[N(1+\delta_i) - \delta_i]} - 2\sigma_i f. \tag{3.32}$$

If firm $n_{-j,i}$ does not defect and thus $M = \{1, .., N\}$ during the action game, its total payoff is

$$\upsilon_{j,i} = \frac{b_i(b_i - 2k)}{2} - 2\sigma_i f. \tag{3.33}$$

Comparing (3.32) and (3.33) we obtain

$$\upsilon_{j,i} > \upsilon_{-j,i} \rightarrow \Pi_i^M = \frac{b_i(b_i - 2k)}{2} - \frac{1}{16}\frac{\Big\{[2N(b_i - k) + (N-1)\delta_i b_i\Big\}^2}{3[N(1+\delta_i) - \delta_i]} > 0. \tag{3.34}$$

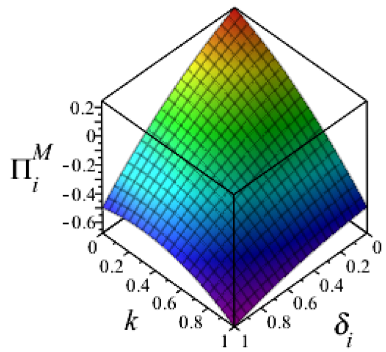
Note that the degree of complexity of internal stability (Π_i^M) analysis in (3.34) increases with N. Figure 9 presents numerical solutions in which the cartel remains internally stable. In Figures 9a, we assume $N = 4$ and $b_i = 1.21$. If $\delta_i \rightarrow 0$ the cartel is stable for $k \leq 0.44$. If $\delta_i \rightarrow 1$, the stable cartel with 4 firms occurs when $k \leq 0.32$. To preserve stable collusion with $N = 4$, when $\delta_i \rightarrow 1$ firms need to be more efficient (lower k). Otherwise, the internal stability does not hold.

In Figure 9b we bring the intuition of how an increase in the demand parameter ($b_i = 2.03$) influences the stability of the cartel with $N = 4$. When $\delta_i \rightarrow 0$, the cartel is stable for $k \leq 0.74$. However, when $\delta_i \rightarrow 1$, $k \leq 0.54$ is required for internal stability. In short, as a positive increment in b_i increases both the cartel payoff and the deviation payoff, stability is guaranteed by a larger range of k values when $\delta_i \rightarrow 0$. This is because only the deviation payoff in equation (3.34) is impacted by δ_i. Conversely, when both k and δ_i tend to one, internal stability is not satisfied. Figure 9c illustrates the conditions under which the cartel remains stable for $N = 5$ and $b_i = 1.21$. When $\delta_i \rightarrow 0$ the cartel is internally stable for $k \leq 0.348$. Considering $\delta_i \rightarrow 1$, the stable cartel occurs for all $k \leq 0.149$.

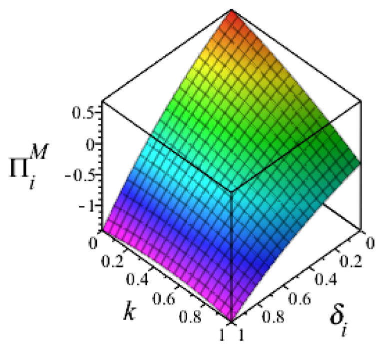
In Figure 9d we offer an intuition regarding how an increase in the demand parameter ($b_i = 2.03$) influences the stability of the cartel with $N = 5$. Note that when $\delta_i \rightarrow 0$, the cartel is stable with 5 firms if $k \leq 0.58$. However, when $\delta_i \rightarrow 1$, $k \leq 0.25$ is required for internal stability. In summary, as a positive increment in b_i increases both the cartel payoff and the deviation payoff, stability is guaranteed by a larger range of k values when $\delta_i \rightarrow 0$. As stated before, only the deviation payoff is impacted by δ_i.

Finally, we present some boundary cases in Figures 9e and 9f, respectively. Note that when both k and δ_i tend to zero, it is possible to find an internal stability solution for the cartel with $N = 6$. In other words, for this extreme case, firms would offer independent goods and have the most efficient production technology.

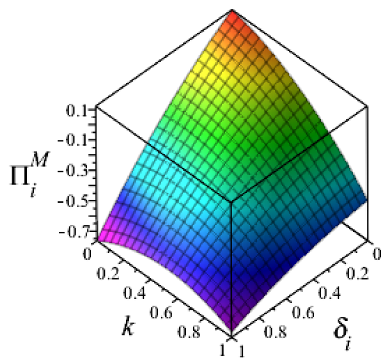
Figure 9 – Internal stability Π_i^M according to N, k, δ_i and b_i.



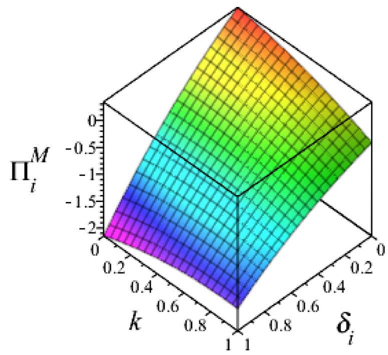
(a) $b_i = 1.21; N = 4$



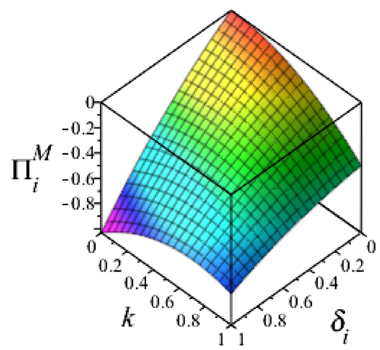
(b) $b_i = 2.03; N = 4$



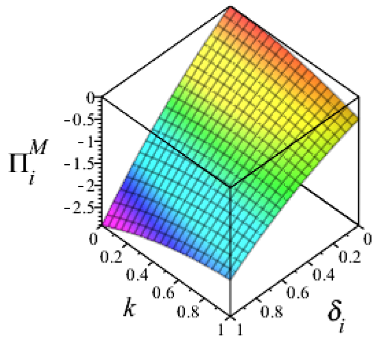
(c) $b_i = 1.21; N = 5$



(d) $b_i = 2.03; N = 5$



(e) $b_i = 1.21; N = 6$



(f) $b_i = 2.03; N = 6$

Source: Elaborated by the authors.

There is still a last boundary situation, where firms have very low production technology ($k \to 1$), but this inefficiency is offset if $b_i \to \infty$. However, this last case is rather unrealistic, and the cartel is weakly internally stable, i.e., $\Pi_i^M \leq 0$. Given the analysis of both external and internal stability for the homogeneous case, we can summarize the findings in Result 2, as follows:

Result 2. For the case where $b_i = 1.21$ and the degree of product differentiation $\delta_i \to 0$, the cartel remains stable with $N = 4(N = 5)$ if $k \leq 0.44(k \leq 0.348)$. There is also a boundary situation in which the stability holds for a collusion made up with 6 homogeneous firms as long as $k = 0$. On the other hand, when $\delta_i \to 1$, the cartel remains stable with 4(5) firms for all $k \leq 0.32(K \leq 0.149)$. Faced with a growth in the demand parameter $b_i = 2.03$, but considering $\delta_i \to 0$, the collusion is stable with four (five) firms for $k \leq 0.74(k \leq 0.58)$. When $\delta_i \to 1$, the stability for $N = 4(N = 5)$ firms is given by $k \leq 0.54(k \leq 0.25)$.

3.3.7 The setting with heterogeneous firms

Now, we consider an oligopolistic industry in a region i consisting of $n_{j,i} \geq 3$ for $j = 1, 2$ heterogeneous firms. Firms can be low-cost (n_1) and high-cost (n_2), and $n_{1,i} + n_{2,i} = N_i$. In addition, we assume that there are two periods. In each period, if firm j on region i chooses a price $p_{j,i}$ for its product, the quantity demanded of this firm's product is given by equation (3.1). Now, the amount $q_{j,i}$ is supplied by firm j, that can be whether a low-cost or a high-cost firm. Both type of firms face linear cost functions, with $k_{j,i}$ denoting their respective marginal costs, where $b_i > max\{0, k_{j,i}\}$. Considering two type of firms, we allow for cost asymmetry $(k_{1,i} \neq k_{2,i})$. As in the previous model, the antitrust agency is in charge of preventing price fixing (cartelization) among firms in the entire economy. For convenience, we start analyzing the setting in which $n_1 + n_2 = 3$. We assume that there are two high-cost firms $(n_{2,i} = 2)$ and only one low-cost firm $(n_{1,i} = 1)$. We normalize the costs as follows: $k_{1,i} = 0$ and $k_{2,i} = k$, with $k > 0$. In this sense, we assume $p_{1,i} < p_{2,i} = p_{3,i}$. The polar case in which the three firms are honest is available in A and A. In the following subsection we evaluate the case where heterogeneous firms are dishonest.

3.3.8 Cartel prices for the heterogeneous firms

Consider now the other polar case, where all firms are dishonest and belong to the cartel. All the derivations are available in A Hence, $M = D = \{1, 2, 3\}$. In each period, the cartel chooses non negative price $\{p_{1,i}, p_{2,i}, p_{3,i}\}$ to maximize equation (3.7), which yields the following payoffs for the low-cost and high-costs firms, respectively:

$$E\pi_{1,i}^M = \frac{1}{4}b_i^2 + \frac{1}{6}\delta_i b_i k - \sigma_i f, \tag{3.35}$$

$$E\pi_{2,i}^M = \frac{1}{12}(b_i - k)[3(b_i - k) - \delta_i k)] - \sigma_i f. \tag{3.36}$$

3.3.9 Deviation and retaliation in the cartel with heterogeneous firms

We now consider two situations where the cartel contains two firms, say, one low-cost firm and one high-cost firm. We first examine a situation where all firms agree to join the cartel prior to the beginning of the game, but one of the high-cost firms, say, firm 3 deviates in the first period. The cartel observes the deviation and retaliates in the second period. In the retaliation period, the cartel moves first. Retaliation takes the form of stealing some (or all) of the defector's product. Later, we examine a situation where firm 3 decides not to join the cartel prior to the beginning of the game. In this case, the cartel retaliates in each period. Detailed derivations are in A.

Suppose that during pre-game communications, all firms agree to join the cartel. During the action game, however, firm 3 deviates. Hence, $D = \{1, 2, 3\}$ and $M = \{1, 2\}$.

Consider the first period. Firms 1 and 2 set their cartel prices. Then, firm 3 chooses non-negative $p_{3,i}$ to maximize

$$\pi_{3,i} = \frac{1}{3}(p_{3,i} - k)[3b_i + \delta_i P_{-3,i} - p_{3,i}(3 + 2\delta_i)]. \tag{3.37}$$

Thus, firm 3's expected payoff in the first period is

$$E\pi_{3,i} = \frac{[2(3 + \delta_i)b_i - 3(2 + \delta_i)k]^2}{48(3 + 2\delta_i)} - \sigma_i f. \tag{3.38}$$

Each cartel member earns the following expected payoff in the first period:

$$\begin{aligned} E\pi^M_{1,i} &= \frac{[2(9 + 6\delta_i - \delta_i^2)b_i + 3\delta_i(4 + 3\delta_i)k]b_i}{24(3 + 2\delta_i)} - \sigma_i f, \\ E\pi^M_{2,i} &= \frac{(b_i - k)[2(9 + 6\delta_i - \delta_i^2)b_i - 3(6 + 6\delta_i + \delta_i^2)k]}{24(3 + 2\delta_i)} - \sigma_i f. \end{aligned} \tag{3.39}$$

As in the previous (symmetric) case, the cartel moves first in the second period and retaliates. The cartel steals a quantity s from the defector and sets its price knowing how the defector will react. Again, We assume that the cartel faces a cost $c \in (0, b_i)$ per unit of quantity stolen from the defector. The defector observes $\{p_{1,i}, p_{2,i}, s\}$ and chooses non-negative $p_{3,i}$ to maximize

$$\pi_{3,i} = \frac{1}{3}(p_{3,i} - k)[3b_i + \delta_i P_{-3,i} - p_{3,i}(3 + 2\delta_i) - s]. \tag{3.40}$$

The cartel anticipates how the defectors will react and choose non-negative $\{p_{1,i}, p_{2,i}, s\}$ to maximize

$$\begin{aligned} &\frac{1}{3}\Bigg\{p_{1,i}[3b_i + \delta_i P_{-1,i} + \frac{s}{2} - p_{1,i}(3 + 2\delta_i)] + (p_{2,i} - k)[3b_i + \delta_i P_{-2,i} + \\ &\frac{s}{2} - p_{2,i}(3 + 2\delta_i)]\Bigg\} - cs \\ \text{s.t.} \quad & s \leq 3b_i + \delta_i P_{-3,i} - (3 + 2\delta_i)k \end{aligned} \tag{3.41}$$

The constraint follows from equation (3.40) and the fact that the quantity sold by firm 3 cannot be negative. Since the objective function (3.41) is linear in s, we obtain

$$s = 3b_i + \delta_i P_{-3,i} - (3 + 2\delta_i)k \quad if \quad p_{1,i} + p_{2,i} \geq 6c, \tag{3.42}$$

$$s = 0 \quad if \quad p_{1,i} + p_{2,i} < 6c. \tag{3.43}$$

Assume initially that the inequality in (3.42) holds. Thus, $p_{3,i} = q_{3,i} = 0$ and the cartel chooses non-negative $\{p_{1,i}, p_{2,i}\}$ to maximize

$$\begin{aligned} &\frac{1}{2}\Bigg\{p_{1,i}[3b_i + \delta_i p_{2,i} - p_{1,i}(2 + \delta_i)] + (p_{2,i} - k)[3b_i + \delta_i p1, i - p_{2,i}(2 + \delta_i)]\Bigg\} \\ &- c[3b_i + \delta_i(p_{1,i} + p_{2,i}) - (3 + 2\delta_i)k]. \end{aligned} \tag{3.44}$$

From (3.42) we can find the values of s and c, respectively:

$$s = \frac{3(b_i - k)(2 + \delta_i)}{2} - \delta_i^2 c. \tag{3.45}$$

$$c \leq \frac{3b_i + k}{2(6 + \delta_i)}. \tag{3.46}$$

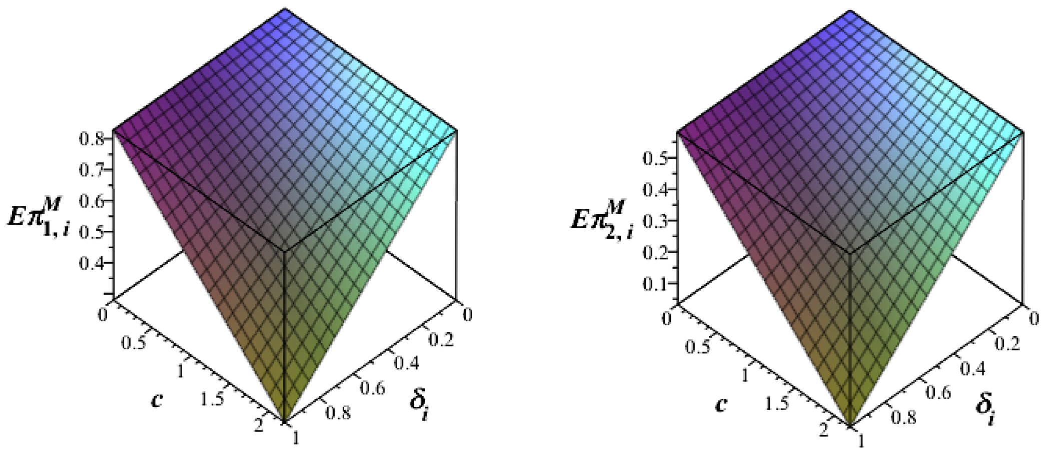
Inequality (3.46) is the necessary condition for the cartel to steal firm 3's product. Consequently, each cartel member's expected payoff is given by

$$\begin{aligned} E\pi_{1,i} &= \frac{1}{8}(3b_i - 2\delta_i c)[3b_i + 2\delta_i c - (2 + \delta_i)k] - \sigma_i f; \\ E\pi_{2,i} &= \frac{1}{8}[3b_i - 2(\delta_i c + k)][3b_i + 2\delta_i c - (2 + \delta_i)k] - \sigma_i f. \end{aligned} \tag{3.47}$$

The expected payoff earned by firm 3 is

$$E\pi_{3,i} = -\sigma_i f. \tag{3.48}$$

Figure 10 – Stability for heterogeneous firms according to c and δ_i when $b_i = 2.21; k \to 1$.



Source: Elaborated by the authors.

As in Subsection 3.3.2, it is trivial to show that when the inequality in (3.46) holds the cartel will always be stable. However, given that firms now are heterogeneous, we offer a comparative statics showing how the cost of stealing c affects each cartel member's expected payoff as in (3.47). It is straightforward to see that, as before, the lower c the greater $E\pi_i^M$. On the other hand, note that given the cost asymmetry between firms, when we compare inequalities (3.15) with (3.46), we see that - everything else constant - the cost of stealing is lower when the market is made up of heterogeneous firms. As well, Figure 10 illustrates how the stability holds for both the low and high-cost firms even when $k \to 1$. As before, s is linear in the objective function (3.41). Thus, this comparative statics holds even if there are N heterogeneous firms in the market - as we evaluate in Subsection 3.3.10.

Suppose now that the inequality in (3.43) holds. Then, the cartel chooses non-negative $\{p_{1,i}, p_{2,i}\}$ to maximize

$$\frac{1}{3}\left\{\sum_{F=1}^{2}(p_{F,i}-k_{F,i})\left[3b_i+\delta_i(p_{3,i}+p_{-F,i})-p_{F,i}(3+2\delta_i)\right]\right\}, \tag{3.49}$$

where $k_{1,i}=0$, $k_{2,i}=k$, $p_{-F,i}=p_{2,i}^{M}$ if $F=1$ and $p_{-F,i}=p_{1,i}^{M}$ if $F=2$. The expected payoff earned by each cartel member is

$$E\pi_{1,i}^{M}=\frac{1}{48}\frac{\left\{2[3(6+5\delta_i)+\delta_i^2]b_i+\delta_i(18+13\delta_i)k\right\}\left\{2[3(6+5\delta_i)+\delta_i^2]b_i+\delta_i(6+5\delta_i)k\right\}}{4(3+2\delta_i)^2(3+\delta_i)}-\sigma_i f, \tag{3.50}$$

$$E\pi_{2,i}^{M}=\frac{1}{48}\frac{\left\{2[3(6+5\delta_i)+\delta_i^2]b_i-[6(6+7\delta_i)+11\delta_i^2]k\right\}\left\{2[3(6+5\delta_i)+\delta_i^2]b_i-3[2(6+5\delta_i)+\delta_i^2]k\right\}}{4(3+2\delta_i)^2(3+\delta_i)}-\sigma_i f, \tag{3.51}$$

Firm 3's expected payoff is

$$E\pi_{3,i}=\frac{1}{48}\frac{\left\{2[\delta_i^3+9(6+8\delta_i+3\delta_i^2)]b_i-[7\delta_i^3+18(6+9\delta_i+4\delta_i^2)]k\right\}^2}{4(3+2\delta_i)^2(3+\delta_i)}-\sigma_i f. \tag{3.52}$$

The last case to consider is the one in which during the pre-game communications firms 1 and 2 decide to form the cartel, while firm 3 decides to stay out. Hence, $D=M=\{1,2\}$ and $H=\{3\}$. In this situation, the cartel moves first and retaliates against firm 3 in both periods. The game played in each period is identical to the game that the cartel and firm 3 play in the retaliation period in the case examined just before this one. Hence, if condition (3.42) holds, equation (3.47) is the expected payoff per period for each cartel member. The payoff per period for firm 3 is $\pi_{3,i}=0$.

if condition (3.42) does not hold, equations (3.50) and (3.51) are the expected payoff per period for the low and the high-cost firms in the cartel, respectively. The payoff per period for firm 3 is

$$\pi_{3,i}=\frac{1}{48}\frac{\left\{2[\delta_i^3+9(6+8\delta_i+3\delta_i^2)]b_i-[7\delta_i^3+18(6+9\delta_i+4\delta_i^2)]k\right\}^2}{4(3+2\delta_i)^2(3+\delta_i)}. \tag{3.53}$$

Employing the concept of internal and external stability to characterize a stable cartel, we now show the conditions under which $M=\{1,2,3\}$ is stable. By definition, this set is externally stable. Hence, we only need to establish the conditions under which it is also internally stable. Assume throughout that the payoff in period 2 is valued at the same rate as the payoff in period 1; that is, the inter-temporal discount rate is zero.

If $M = \{1, 2, 3\}$ is formed during pregame communications, a unilateral deviation during the action game takes us to the setting in which one firm deviates in the first period and the other two firms, which remain in the cartel, retaliate in the second period. As before, let firm 3 be the defector. Thus, $M = \{1, 2\}$ during the action game. Assume that condition (3.42) holds. Firm 3's total payoff is as follows when it defects

$$\mathcal{V}_{3,i} = \frac{[2(3+\delta_i)b_i - 3(2+\delta_i)k]^2}{48(3+2\delta_i)} - 2\sigma_i f, \tag{3.54}$$

If firm 3 does not defect and thus $M = \{1, 2, 3\}$ during the action game, it earns

$$\mathcal{V}_i^M = \frac{1}{6}(b_i - k)[3(b_i - k) - \delta_i k)] - 2\sigma_i f. \tag{3.55}$$

Comparing (3.54) and (3.55), we obtain

$$\mathcal{V}_i^M - \mathcal{V}_{3,i} > 0 \to \frac{1}{12}(b - k)[(9 + 6\delta_i - \delta_i^2)b_i - (9 + 9\delta_i + \delta_i^2)k] > 0. \tag{3.56}$$

We can simplify the necessary condition in equation (3.56) as follows: $\Phi b_i > \Theta k$, where

$$\Phi(\delta_i) = \frac{9 + \delta_i(6 - \delta_i)}{12}, \quad \Theta(\delta_i) = \frac{9 + \delta_i(9 + 2\delta_i)}{12}.$$

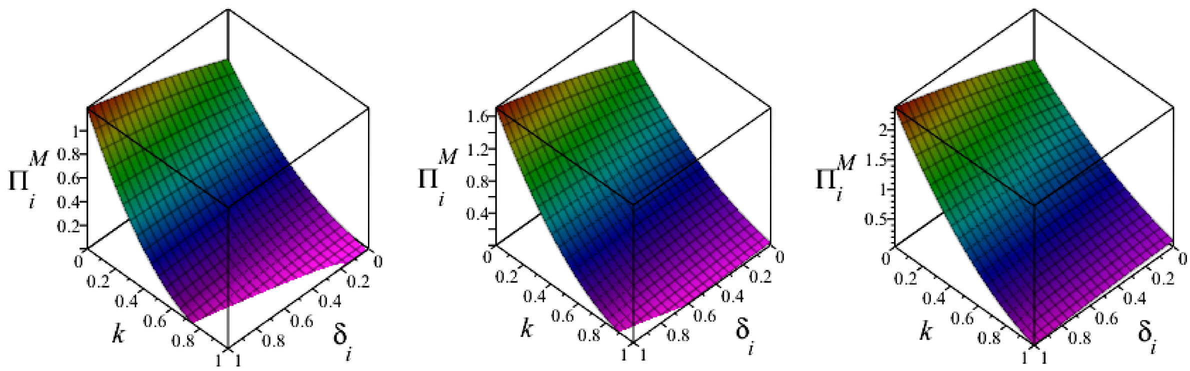
Result 3 below provides a summary of the findings on cartel stability considering heterogeneous firms.

Result 3. Assuming that $\delta_i = 0$, it is easy to see that $\mathcal{V}_i^M - \mathcal{V}_{3,i}$ is positive because by definition $b_i > k$. Hence, firm 3 has no incentive to deviate from the cartel during the action game. It follows that $M = \{1, 2, 3\}$ is stable if condition (3.46) holds. Figure 18 illustrates through a numerical example under which conditions the internal stability of the cartel is sustained in the heterogeneous case. Note that as b_i increases, the larger the difference between Φb_i and Θk. Figure 18a is a useful benchmark as it illustrates the condition where b_i is slightly above the upper bound of k. When $\delta_i \to 0$, the cartel remains stable even with $k \approx 1$. From Figure 18b we observe that the cartel is internally stable if $k \in [0, 0.85)$. Figure 11c illustrates how an increase in the value of b_i contributes to the internal stability of the cartel relative to k when $\delta_i = 1$. Note that there is no incentive for firm 3 to deviate from the collusive agreement if $b_i \approx 1.43$ and the cartel is internally stable for all $k \in [0, 1]$. Thus, comparing Figures 18 with Figure 17 we observe that the cartel in the heterogeneous case requires a lower value b_i to guarantee the internal stability.

3.3.10 The setting with N heterogeneous firms

Now consider the case with N_i heterogeneous firms in region i, that is, $N_{l,i} + N_{h,i} + = N_i$. Where $N_{l,i} = \sum_{l=1}^{L} n_{l,i}$ and $N_{h,i} = \sum_{h=1}^{H} n_{h,i}$ represents the number of low-cost and

Figure 11 – Internal stability according to δ_i, b_i and k.



(a) $b_i = 1.01$ (b) $b_i = 1.21$ (c) $b_i = 1.43$

Source: Elaborated by the authors.

high-cost firms, respectively. We normalize to zero the costs associated to the low-cost firms. The cost of the high-cost type firm is given by $k \in [0,1]$ with $b_i > max\{0, k\}$. For the case where all firms are dishonest and belong to the cartel, we have $M = D = \{N_{l,i} + N_{h,i}\}$. In each period, the cartel chooses non negative $\{p_{l,i}, p_{h,i}\}$ to maximize the expected payoff, taking σ_i and f as given:

$$\begin{aligned}\sum_{j=1}^{N} E\pi_{j,i}^M =& \sum_{j=1}^{N} \Bigg\{ \frac{N_l}{N}(p_{l,i})[Nb_i + \delta_i P_{-l,i} - p_{l,i}(N + (N-1)\delta_i)] \\ &+ \frac{N_h}{N}(p_{h,i} - k)[Nb_i + \delta_i P_{-h,i} - p_{h,i}(N + (N-1)\delta_i) - \sigma_i f \Bigg\}.\end{aligned} \tag{3.57}$$

Each firm earn the following expected payoff in each period:

$$\begin{aligned}E\pi_{l,i}^M =& \frac{1}{4}\frac{b_i(Nb_i + N_{h,i}\delta_i k)}{N} - \sigma_i f; \\ E\pi_{h,i}^M =& \frac{1}{4}\frac{(b_i - k)\Big\{[(b_i - (1+\delta_i)k)]N + N_{h,i}\delta_i k\Big\}}{N} - \sigma_i f.\end{aligned} \tag{3.58}$$

Unlike the homogeneous case, here the number of high-cost firms $N_{h,i}$ in the cartel directly impacts the payoff of the firms. We now consider a situation where the cartel contains $N_i - 1$ firms. Suppose that during the pre-game communications, all N firms agree to join the cartel. During the action game, however, firm $n_{-j,i}$, which can be either high-cost or low-cost, deviates. deviates. Hence, $D = \{n_{1,i}, ...N_{i,j}\}$ and $M = \{n_{1,i}...N - 1_{j,i}\}$. First, let us consider the case where a high-cost firm deviates. Next, we evaluate the case where a low-cost firm cheats on the cartel.

3.3.11 High-cost firm deviates

In this case, firms in the cartel set the following prices:

$$p_{l,i}^M = \frac{b_i}{2}; \qquad p_{h,i}^M = \frac{b_i + k}{2}. \tag{3.59}$$

By it turn, the high-cost deviant firm $n^D_{h,i}$ takes (3.59) into account and then chooses non-negative $p^D_{h,i}$ to maximize:

$$\pi^D_{h,i} = \frac{1}{N}(p^D_{h,i} - k)\Big\{Nb_i + P^M_{-h}\delta_i - p^D_{h,i}[N + (N-1)\delta_i]\Big\}. \tag{3.60}$$

Where $P^M_{-h} = p^M_{l,i}N_{l,i} + p^M_{h,i}(N_{h,i} - 1)$. Hence, firm $n^D_{h,i}$'s expected payoff in the first period is

$$\pi^D_{h,i} = \frac{1}{16}\frac{\Big\{[(N_{h,i} - N_{l,i} - 1)b_i + (2N + N_{h,i} - 3)k]\delta_i - 2N(b_i - k)\Big\}^2}{N[N + (N-1)\delta_i]} - \sigma_i f. \tag{3.61}$$

We now must employ the concept of external and internal stability to characterize a stable cartel. We start with the external stability analysis.

3.3.12 External stability for high-cost firms

In this case, the cartel is externally stable if it is not profitable for a high-cost fringe firm to join the collusion. Thus, from equations (3.58) and (3.61) we derive the following condition to guarantee the external stability ($\Pi^D_{h,i}$) for a high-cost firm:

$$\Pi^D_{h,i} \geq 0, \quad where \quad \Pi^D_{h,i} = \pi^D_{h,i} - \pi^M_{h,i}. \tag{3.62}$$

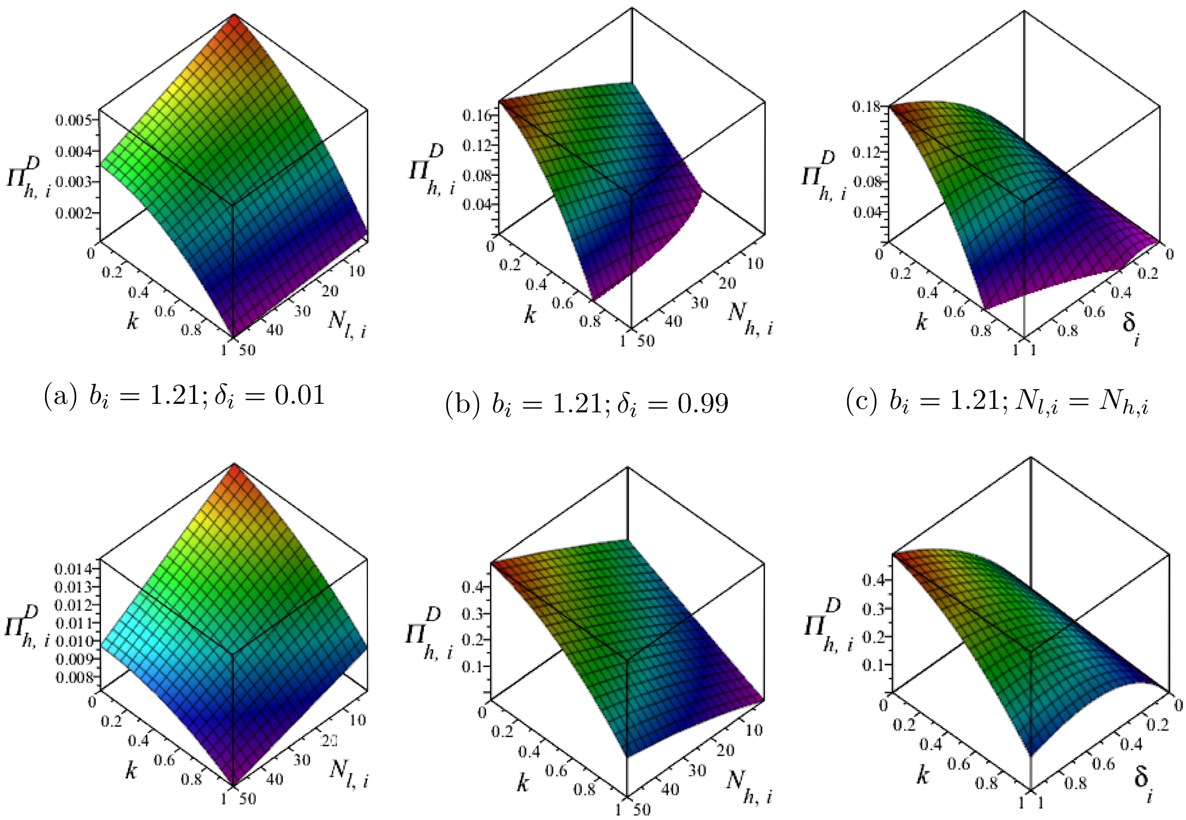
It is straightforward to see that when $\delta_i \to 0$, both equations (3.58) and (3.61) becomes equal to $\frac{(b_i-k)^2}{4} - \sigma_i f$, and the cartel is weakly external stable, i.e., $\Pi^D_{h,i} = 0$. On the other hand, when $\delta_i \to 1$ and assuming that $Nh, i = Nl, i = N/2$, the payoff in (3.58) is equal to $E\pi^M_{h,i} = \frac{1}{8}(b_i - k)(2b_i - 3k) - \sigma_i f$. The expression (3.61) becomes equal to $\pi^D_{h,i} = \frac{[(5N/2-3)k - b_i - 2N(b_i-k)]^2}{16N(2N-1)} - \sigma_i f$. Canceling $\sigma_i f$ in both expressions, if $b_i \geq 3k/2$, then $E\pi^M_{h,i} \geq 0$. By setting $b_i = 3k/2$ we have:

$$\pi^D_{h,i} = \frac{(-Nb_i - 3b_i)^2}{16N(2N-1)} > 0, \quad E\pi^M_{h,i} = 0 \quad and \quad \Pi^D_{h,i} > 0.$$

Besides these polar cases, Figure 12 shows numerical solutions to another situations in which the condition (3.62) holds. We consider a market made up of 1/2 high-cost firms and 1/2 low-cost firms ($N_{h,i} = N_{l,i} = 50$). From Figures 12a to 12c we consider $b_i = 1.21$ and derive the following patterns in relation to firm behavior. When $\delta_i \to 0$, external stability is decreasing in both the number of low-cost firms and parameter k. On the other hand, although we have omitted the illustrations, external stability is increasing in the number of high-cost firms, but remains decreasing in the level of asymmetry k. From Figure 12b, as $\delta_i \to 1$, we see that $\Pi^D_{h,i}$ is not satisfied for all values of $k \in (0.1)$.

Note that it is also increasing in $N_{h,i}$, but decreasing in k. In summary, when $k \to 1$, external stability is guaranteed for $\delta_i \leq 0.281$. When $\delta_i \to 1$, external stability

Figure 12 – External stability according to $N_{l,i}$, $N_{h,i}$, b_i; δ_i and k.



(a) $b_i = 1.21; \delta_i = 0.01$ (b) $b_i = 1.21; \delta_i = 0.99$ (c) $b_i = 1.21; N_{l,i} = N_{h,i}$

(d) $b_i = 2.0; \delta_i = 0.01$ (e) $b_i = 2.0; \delta_i = 0.99$ (f) $b_i = 2.0; N_{l,i} = N_{h,i}$

Source: Elaborated by the authors.

hold as long as $k \leq 0.715$. Assuming a value $b_i = 2.0$ we can find new conclusions about the pattern governing external stability when looking at Figures 12d, 12e and 12f. Note that although $\Pi^D_{h,i}$ continues decreasing (increasing) in $N_{l,i}$ ($N_{h,i}$), it now holds for the entire range of k and δ_i. We highlight that the patterns discussed here hold even when we consider a market made up with 2/3 of high-cost firms.

3.3.13 Internal stability for high-cost firms

If $M = \{1, .., N_i\}$ is formed during pre-game communications, a unilateral deviation of a high-cost firm during the action games takes us to the setting in which one firm deviates in the first period and the other $N_i - 1$ firms, which remain in the cartel, retaliate in the second period. As before, let firm $n^D_{h,i}$ be the defector. Thus, $M = \{1, .., N_i\}$ during the action game. Assume that the following condition holds:

$$s = N[b_i - (1 + \delta_i)k] + P^M_{-h}\delta_i + \delta_i k \quad if \quad P^M_{-h} \geq 2Nc. \tag{3.63}$$

In this case, firm $n^D_{h,i}$'s total payoff is as follows when it defects

$$\mathcal{V}^D_{h,i} = \frac{1}{16}\frac{\Big\{[(N_{h,i}-N_{l,i}-1)b_i + (2N+N_{h,i}-3)k]\delta_i - 2N(b_i-k)\Big\}^2}{N[N+(N-1)\delta_i]} - 2\sigma_i f. \qquad (3.64)$$

If firm $n_{-j,i}$ does not defect and thus $M = \{1, .., N\}$ during the action game, its total payoff is

$$\mathcal{V}^M_{h,i} = \frac{1}{2}\frac{(b_i-k)\Big\{[b_i-(1+\delta_i)k]N + N_{h,i}\delta_i k\Big\}}{N} - 2\sigma_i f. \qquad (3.65)$$

Comparing (3.64) and (3.65) the cartel is internally stable if $\mathcal{V}^M_{h,i} \geq \mathcal{V}^D_{h,i}$:

$$\frac{1}{2}\frac{(b_i-k)\Big\{[b_i-(1+\delta_i)k]N + N_{h,i}\delta_i k\Big\}}{N} \geq \frac{1}{16}\frac{\Big\{[(N_{h,i}-N_{l,i}-1)b_i + (2N+N_{h,i}-3)k]\delta_i - 2N(b_i-k)\Big\}^2}{N[N+(N-1)\delta_i]}. \qquad (3.66)$$

Suppose initially that $\delta_i \to 0$. The internal stability of the cartel derived in (3.66) can be reduced to:

$$b_i \geq k.$$

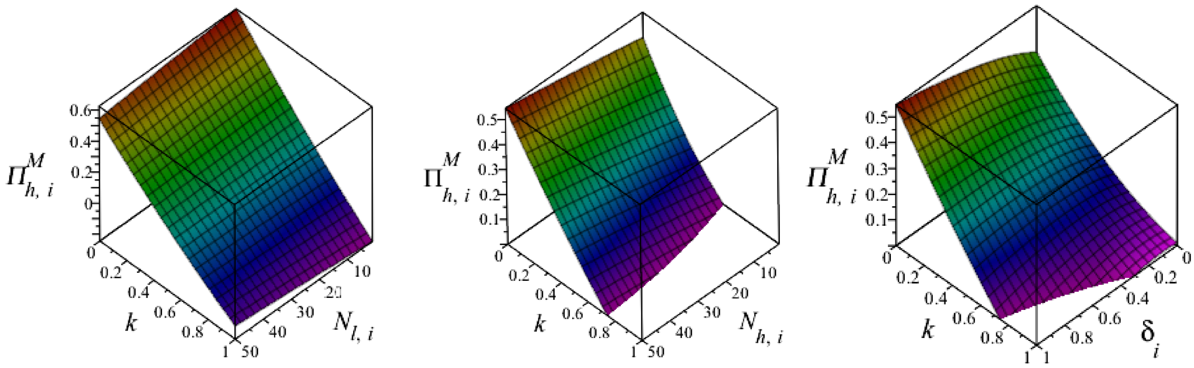
As we define $b_i > k$, it is straightforward to show that the cartel is internally stable. To assess internal stability when $\delta_i \to 1$, assume the following assumptions:
(i) $b_i \geq 2k$;
(ii) $N_{h,i} \geq N_{l,i}$.
Considering the case where the equality holds in both (i) and (ii), we have

$$\Pi^M_{h,i} \geq 0 \to \mathcal{V}^M_{h,i} \geq \mathcal{V}^D_{h,i} \to 8N_{h,i}(4N_{h,i}-1) \geq (N_{h,i}-5)^2.$$

Which always hold when $N_{h,i} \geq 1$. Note also that under these conditions internal stability is increasing in the number of high-cost firms. We now evaluate through the illustration in Figure 13 the internal stability of the cartel by assuming $k < b_i < 2k$. For simplicity, we assume that $b_i = 1.21$ and $N_{l,i} = N_{h,i}$.

In Figure 13a, we see that for a low level of asymmetry between firms, internal stability is decreasing in the number of low-cost firms that join the collusion. However, as $k \to 1$, the cartel's internal stability is increasing in $N_{l,i}$. In turn, in Figure 13b, we see that the internal stability is increasing as $k \to 1$. As this degree of asymmetry increases, and preserving $N_{l,i} = N_{h,i}$, the cartel remains stable for $k \leq 0.741$. Figure 13c shows the relationship between internal stability with both parameters k and δ_i. Note that when $K = 0$, $\Pi^M_{h,i}$ is increasing with δ_i. As k increases, $\Pi^M_{h,i}$ decreases in both cost asymmetry and product differentiation. In summary, we have:
(iii) If $k \to 1, \Pi^M_{h,i} \geq 0$ for $\delta_i \leq 0.33$;
(iv) If $\delta_i \to 1, \Pi^M_{h,i} \geq 0$ for $k \leq 0.741$.

Figure 13 – $\Pi^M_{h,i}$ according to $N_{l,i}$, $N_{h,i}$ and k.



(a) $b_i = 1.21; \delta_i = 0.99$ (b) $b_i = 1.21; \delta_i = 0.99$ (c) $b_i = 1.21; N_{l,i} = N_{h,i}$

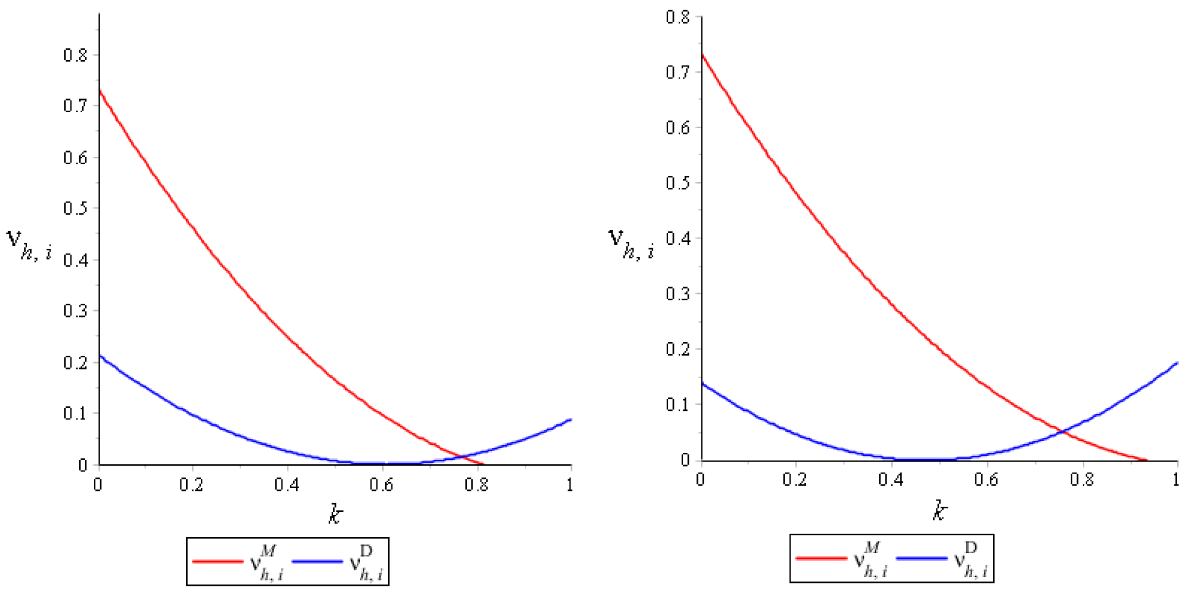
Source: Elaborated by the authors.

Figures 14a and 14b complement the intuition regarding the increase in the proportion of high-cost firms in the collusive arrangement, i.e., $N_{h,i} = 2/3$. In other words, the cartel's internal stability is more sensitive to the entry of high-cost firms and, for $\Pi^M_{h,i}$ to still holding, we must rewrite conditions (iii) and (iv) as follows:

(v) If $k \to 1, \Pi^M_{h,i} \geq 0$ for $\delta_i \leq 0.30$;

(vi) If $\delta_i \to 1, \Pi^M_{h,i} \geq 0$ for $k \leq 0.731$.

Figure 14 – The internal stability according to k and $N_{h,i}$.



(a) $(\delta_i; b_i; N_{h,i}) = \{0.99; 1.21; 1/2\}$ (b) $(\delta_i; b_i; N_{h,i}) = \{0.99; 1.21; ; 2/3\}$

Source: Elaborated by the authors.

Given the analysis of both external and internal stability, we can summarize the findings in Result 4, as follows:

Result 4. Considering $b_i \geq 2$ and $N_{h,i} \geq N_{l,i}$ the cartel is always stable. By setting $b_i < 2$ and $N_{h,i} = \frac{1}{2}N$, if $\delta_i \to 1$ the stability holds for $k \leq 0.715$. As $k \to 1$, the cartel is stable for $\delta_i \leq 0.281$. The collusion remains stable for a market made up with $N_{h,i} = \frac{2}{3}N$ high-cost firms.

3.3.14 Low-cost firm deviates

In this case, the low-cost deviant firm $n^D_{l,i}$ takes (3.57) into account and then chooses non-negative $p^D_{l,i}$ to maximize:

$$\pi^D_{l,i} = \frac{1}{N} p^D_{l,i} \Big\{ N b_i + P^M_{-l} \delta_i - p^D_{l,i}[N + (N-1)\delta_i] \Big\}. \tag{3.67}$$

Where $P^M_{-l} = p^M_{l,i}(N_{l,i} - 1) + p^M_{h,i} N_{h,i}$. Hence, firm $n^D_{l,i}$'s expected payoff in the first period is

$$\pi^D_{l,i} = \frac{1}{16} \frac{\Big\{ [2N + (N_{h,i} - N_{l,i} + 1)\delta_i] b_i + N_{h,i}\delta_i k \Big\}^2}{N[N + (N-1)\delta_i]} - \sigma_i f. \tag{3.68}$$

3.3.15 External stability for low-cost firms

In this case, the cartel is externally stable if it is not profitable for a low-cost fringe firm to join the collusion. Thus, from equations (3.58) and (3.68) we derive the following condition to guarantee the external stability ($\Pi^D_{l,i}$) for a low-cost firm:

$$\Pi^D_{l,i} \geq 0, \quad where \quad \Pi^D_{l,i} = \pi^D_{l,i} - \pi^M_{l,i}. \tag{3.69}$$

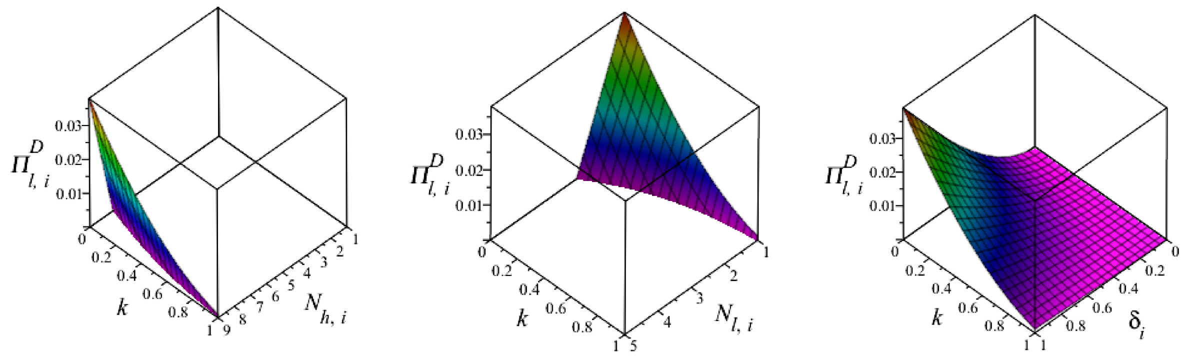
It is straightforward to see that when $\delta_i \to 0$, both equations (3.58) and (3.68) becomes equal to $\frac{(b_i - k)^2}{4} - \sigma_i f$, and the cartel is weakly external stable, i.e., $\Pi^D_{h,i} = 0$. On the other hand, when $\delta_i \to 1$ the analysis needs to be more careful. Therefore, in Figure 15 we show a numerical solution to evaluate the external stability.

Unlike the case where the high-cost firm deviates, external stability for the low-cost firm occurs for the interval $\delta_i \in (0,1)$ only when $N_{l,i} = \frac{1}{10}N$. For simplicity, we consider in Figure 15 a market in which there are only 1 low-cost and 9 high-cost firms. Assuming $b_i = 1.21$ and $\delta_i \to 1$, we can see in Figures 15a and 15b that the external stability is decreasing both in the cost asymmetry and in $N_{l,i}$, respectively. Figure 15c illustrates the relationship between k and δ_i. Note that when $k \to 0$, $\Pi^D_{l,i}$ increases in δ_i. We emphasize that, considering a market with 8 high-cost and 2 low-cost firms, $\Pi^D_{l,i}$ does not hold for any $\delta_i > 0$ and $b_i \geq 1.21$.

3.3.16 Internal stability for low-cost firms

We now must employ the concepts of internal stability to characterize a stable cartel. If $M = \{1, .., N_i\}$ is formed during pre-game communications, a unilateral deviation

Figure 15 – $\Pi^D_{l,i}$ according to$N_{l,i}, N_{h,i}, \delta_i$ and k for $\delta_i = 0.99; b_i = 1.21; N_{l,i} = 1; N_{h,i} = 9.$



(a) $N_{l,i} = \frac{1}{10}N$ (b) $N_{h,i} = \frac{9}{10}N$ (c) $N_{l,i} = \frac{1}{10}N$

Source: Elaborated by the authors.

of a low-cost firm during the action games takes us to the setting in which one firm deviates in the first period and the other $N_i - 1$ firms, which remain in the cartel, retaliate in the second period. As before, let firm $n^D_{l,i}$ be the defector. Thus, $M = \{1, .., N_i\}$ during the action game. Assume that the following condition holds:

$$s = N[b_i - (1+\delta_i)k] + P^M_{-l}\delta_i + \delta_i k \quad if \quad P^M_{-l} \geq 2Nc. \tag{3.70}$$

In this case, firm $n^D_{l,i}$'s total payoff is as follows when it defects

$$\mathcal{V}^D_{l,i} = \frac{1}{16}\frac{\left\{[2N + (N_{h,i} - N_{l,i} + 1)\delta_i]b_i + N_{h,i}\delta_i k\right\}^2}{N[N + (N-1)\delta_i]} - 2\sigma_i f. \tag{3.71}$$

If firm $n_{l,i}$ does not defect and thus $M = \{1, .., N\}$ during the action game, its total payoff is

$$\mathcal{V}^M_{l,i} = \frac{1}{2}\frac{b_i(Nb_i + N_{h,i}\delta_i k)}{N} - 2\sigma_i f. \tag{3.72}$$

Comparing (3.71) and (3.72) the cartel is internally stable if:

$$8[N + (N-1)\delta_i](Nb_i + N_{h,i}\delta_i k)b_i \geq \left\{[2N + (N_{h,i} - N_{l,i} + 1)\delta_i]b_i + N_{h,i}\delta_i k\right\}^2. \tag{3.73}$$

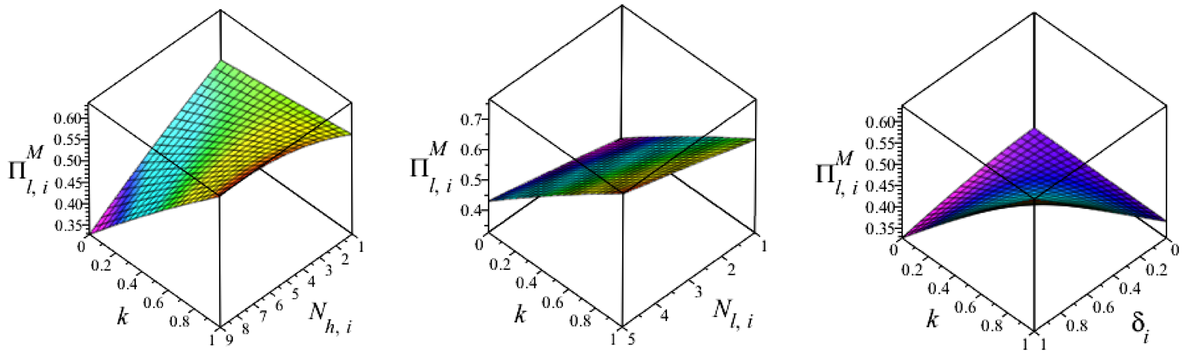
As in the previous case, suppose initially that $\delta_i \to 0$. The relationship that guarantees the internal stability of the cartel derived in (3.73) can be reduced to:

$$2N^2 b_i^2 \geq N^2 b_i^2.$$

It is trivial to deduce that the above condition always hold. On the other hand, when $\delta_i \to 1$, we have that

$$8(2N+1)(Nb_i + N_{h,i}k)b_i \geq \left\{(2N + N_{h,i} - N_{l,i} + 1)b_i + N_{h,i}k\right\}^2.$$

Figure 16 – $\Pi^M_{l,i}$ according to $N_{l,i}, N_{h,i}, \delta_i$ and k for $\delta_i = 0.99; b_i = 1.21; N_{l,i} = 1; N_{h,i} = 9$.



(a) (b) (c)

Source: Elaborated by the authors.

On this matter, we provide numerical solutions in Figure 16 to better assess the internal stability. We focus on the patterns found for the case in which $k < b_i < 2k$. As in the analysis of external stability for the low-cost deviating firm, consider $b_i = 1.21$ and $k \in (0, 1)$, $N_{l,i} = \frac{1}{10}N$ and $N_{h,i} = \frac{9}{10}N$. Figures 16a and 16b shows that internal stability is increasing both in k and in N, respectively. Therefore, $\Pi^M_{l,i}$ holds for $N_{l,i} \geq \frac{1}{10}N$.

Furthermore, Figure 16c shows internal stability considering a market with only 1 low-cost firm and 9 high-cost firms. Note that $\Pi^M_{l,i}$ is increasing both in the cost asymmetry k and in δ_i. Now that we have evaluated both the external and internal stability of the cartel in the case that the low-cost firm deviates, we can state Result 5 as follows.

Result 5. For $b = 1.21$, $N_{h,i} = 9/10$ and $N_{l,i} = 1/10$, there is no incentive for the low-cost firm to deviate from the collusive agreement. Considering these parameters, the cartel remains stable for the entire asymmetry cost range given by $k \in (0, 1)$ and by any degree of product differentiation, given by $\delta_i \in (0.1)$, respectively. Otherwise, as the number of low-cost firms in region i increases, the cartel is no longer externally stable - as the deviation payoff is decreasing in $N_{l,i}$.

3.4 CONCLUSION

Assuming that the cartel behaves like a criminal organization, this paper revisits both the symmetric and the asymmetric oligopoly model with a Bertrand fringe. By considering that the retaliation is illegal and providing a complementary approach between analytical and numerical solutions, we show that the conclusions regarding the size of stable cartels are incomplete. In our model firms sells differentiated goods and our results show that the size of the stable illegal cartel is determined jointly by both the number of high-cost firms in the market and the level of cost asymmetries. Objectively, the cartel's stability increases with $N_{h,i}$ and decreases with k. Moreover, we show that given the low-cost firm's incentive to deviate from the agreement, the cartel remains stable whenever $N_{l,i} = \frac{1}{10}N$. Finally, when we assume an oligopoly with 3 firms, the size of the stable cartel supports full cooperation whenever the parameter b_i is large enough - which makes the parameter k irrelevant for the stability.

Our approach also contributes to the challenges and pitfalls faced by antitrust authorities. To achieve a balance between law and economics, antitrust authorities commonly rely on fine setting methodologies, which albeit different, often involve lengthy assessment procedures and fail to incorporate the tricks and threats of criminal organizations into their theoretical motivations. Once we assume illegal cartel within illegal retaliation strategies, our game-theoretical framework offer a useful review of the key aspects of cartel policies, raising issues of methodological importance in setting optimal cartel fines, and proposing solutions using the economic reasoning of crime. In doing so, we show how economics, law, and antitrust practices find some signs of reconciliation to avoid cartel formation.

Concluding, there are some ways in which this study can be expanded. One way could be to consider a dynamic game with sequentially-rational firms. Another approach could assess how the information asymmetry regarding firms' efficiency could interfere in the optimal fine charged by the regulator. Finally, the aspect of the cartel as a criminal organization could be assessed taking into account a leniency program.

4 MACHINE LEARNING WITH STATISTICAL SCREENS FOR DETECTING CARTELS: AN EVALUATION OF THE BRAZILIAN GASOLINE RETAIL MARKET

ABSTRACT

In this article, we combine machine learning techniques with screens based on the statistical moments of gasoline price distribution for cartel detection and prediction in the Brazilian retail market. In addition to the traditional variance screen, we evaluate how the standard deviation, coefficient of variation, skewness, and kurtosis can be useful features in identifying anti-competitive behavior. To complement our analysis, we evaluate the so-called confusion matrix and discuss trade-offs related to false-positive and false-negative predictions. Our results show that in some cases, false-negative predictions critically increase when the main objective is to minimize false-positive predictions. As well, we offer a discussion regarding the pros and cons of our approach for antitrust authorities aiming at detecting and avoiding gasoline cartels.

keywords: Cartel Screens. Price dynamics. Gasoline retail market. Machine Learning
JEL classification: C21 · C45 · C52 · K40 · L40 · L41

4.1 INTRODUCTION

The discussion about cartel formation is relevant in several markets. Given the persistence of anti-competitive behavior in the industry, the best way to investigate and identify them are relevant issues for the antitrust authorities to enforce competition laws. In an environment where the number of complaints and suspicions of cartels is increasing and, given the restriction of resources available to initiate an investigation, the statistical methods of screening are a useful tool. There is a wide variety of cartel screens that offer to the antitrust agencies practical and efficient detection methods (HARRINGTON, 2008; BOLOTOVA; CONNOR; MILLER, 2008; PERDIGUERO, 2010; ECKERT, 2013; DOANE et al., 2015). One of the key variables used to chart the behavior of gasoline cartels is the retail sales price. It is relatively easy to measure and capable of transmitting information about how the market works. There are several studies following this approach (CONNOR, 2005; ABRANTES-METZ et al., 2006; CHOUINARD; PERLOFF, 2007; NOEL, 2007; ABRANTES-METZ, 2012). The main framework focuses on econometric screens. However, there is no universal consensus on this issue. As well, few studies have evaluated the performance of statistical screens (HUBER; IMHOF, 2019).

Intending to contribute to this discussion, our paper combines machine learning techniques with screens based on statistical moments to identify and predict cartel behavior in the gasoline retail market. Taking the Brazilian market as a case study, we evaluate the out of the sample performance of the proposed methods in a total of 1.920 observations constructed from a weekly database of gasoline sales price in the following cities where collusion was detected: Belo Horizonte[1], Brasília, Caxias do Sul and São Luís. Essentially, we intend to use the history of cases already judged and condemned by the Brazilian competition authority (CADE)[2] for cartel practice (PINHA; BRAGA et al., 2019). The data comprise detected collusion and another of no apparent collusion, i.e., collusion may have occured but it was not detected. To distinguish them properly, we defined a binary cartel classification as a dependent variable. The classification criterion for the cartel period is based on the judgments made by CADE, in which the case records contain the exact period in which the explicit evidence that characterized the collusive agreement in each city was collected. Similarly, the criterion adopted for the classification of the non-cartel period was established following the time when the regulator made public the administrative proceeding against gas stations, as well as the operations to disrupt the gasoline cartels.

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[1] We also consider the municipalities of Betim and Contagem, which make up the metropolitan region of Belo Horizonte and were also involved in the conviction of the cartel.

[2] Administrative Council for Economic Defense. Many legal decisions made by CADE were based on shreds of evidence such as wiretaps, hot documents, text messages, e-mails, etc. Access the following links for details: (i) <https://tinyurl.com/yxz8tgnr> (available in English); (ii) <https://tinyurl.com/y6eoamkp> (available only in Portuguese).

More precisely, we extend the approach proposed by Huber and Imhof (2018), Huber and Imhof (2019) to analyze cartel behavior in the gasoline retail market. In that sense, we propose different machine learning algorithms combined with each of the four statistical moments of the gasoline sales price distribution as a screen. As each screen constructed from the distribution of gasoline prices in each city captures a different aspect of price dynamics, the combination of several different screens opens an avenue for a better understanding of the differences regarding price agreements in the gasoline market. Both Ridge and Lasso regressions rely on logistic models Tibshirani (1996). Random Forest consists of a large number of individual decision trees that operate as an ensemble (HO, 1995; BREIMAN, 2001). Neural Networks are a set of algorithms designed to recognize patterns, and are useful tools for clustering and classifying data (HJORT, 1996; RIPLEY, 2007). Regarding the technical aspects, to implement the algorithms we use both cross-validation and random splitting of the database between the training and test sample. Typically, this is the standard strategy for determining the optimal penalty level both for the Lasso and the Ridge regressors. To parsimoniously assess the trade-off between bias and variance, we repeat these steps 100 times to estimate the accuracy of the classifiers. We define the accuracy as the gap between actual cartels and correctly estimated cartels. In this way, our dependent variable takes a value of 1 if the algorithm classifies the cartel probability greater than or equal to 0.5 and 0 otherwise.

By evaluating the so-called confusion matrix we distinguish the performance of our predictors between false-positive and false-negative predictions (AKOUEMO; POVINELLI, 2016). More specifically, a false-positive classification means that the model tags a price dynamics as a cartel even though no cartel happens. In other words, for the antitrust authority as a regulator and as a policymaker, this error characterizes the worst-case scenario, as it can lead to undue convictions and fines, in addition to wasting resources on unsubstantiated investigations. On the other hand, the false-negative outcome is undesirable as well - once it shows that the algorithm was unable to tag price dynamics as a cartel, even if the cartel happens. Thus, a model that produces many false-negatives can be harmful to the competitive environment. Aware of this, a desirable classification method for the antitrust authority is one capable of categorically balancing the trade-off between false-positive and false-negative outcomes.

Our results provide significant evidence that machine learning techniques are powerful tools for cartel detection in the gasoline retail market. Furthermore, they demonstrate that in certain cases, both skewness and kurtosis are relevant variables to minimize the classification error. On average, the algorithms correctly predict 87% of all evaluated cartels out of the sample. Complementarily, on average, the misclassification rate of the models is 13%. By evaluating the performance of each algorithm, Random Forest, on average, presents the best indicators, incurring a 4% of misclassification rate. Besides, we must emphasize that given the low relative rate of a false-positive and false-negative, the methods proposed in this article proved to be quite effective - even when tightening the rule for classifying price dynamics as a cartel. In other words, the performance of the algorithms remained reasonable even when establishing a threshold higher than the default value of 0.5 to classify our variable of interest as a cartel. This analysis reveals to be interesting for antitrust authorities to establish an optimal trade-off between false-positives and false-negatives by tuning the probability threshold.

Regarding the gasoline market, our recommendations in terms of competition policy are twofold. First, we suggest that the regulator may compute the statistical moments considering the distribution of prices charged in the retail gasoline market. Typically, this information is easily accessible. On the application of this screen in real cases, following the recommendation of Huber and Imhof (2019), the classification rule should be adjusted to a threshold between 0.5 and 0.7. Second, with the history of cases already judged and condemned by the antitrust authority, it is possible to associate the price dynamics in a certain period with the cartel behavior. In this way, we provide a sample training to our algorithm to make out of sample predictions. Finally, the regulator can use this approach as a useful tool in the investigation of suspicious markets and firms to assess whether the market practices fit into cartel behavior.

To develop this discussion, the remainder of this paper is organized as follows. Section 4.2 reviews the literature on implementing screens to detect cartels in the gasoline retail market. Section 4.3 describes our data that includes four gasoline cartel cases in the Brazilian fuel retail market and discusses the screens used as predictors for detecting collusive market behavior. Section 4.4 presents the machine learning techniques and Section 4.5 discusses the empirical results. Section 4.6 discusses several policy implications regarding the machine learning algorithms combined with statistical moments screen. Section 4.7 concludes.

4.2 LITERATURE REVIEW

There are two categories of cartel screens: Structural screens identify markets that are likely to be subject to cartelization due to industry characteristics. Behavioral screens detect cartels by detecting patterns in market outcomes that are treated as signs of collusion (HARRINGTON, 2008; ABRANTES-METZ, 2012; CREDE, 2019). The literature on behavioral cartel screens has grown significantly in the last decade. Most notable are the contributions of Abrantes-Metz et al. (2006), Bolotova, Connor and Miller (2008) who propose cartel screens that are based on the analysis of price variance in an industry. Most behavioral screens so far have been specifically tailored to detect bid-rigging conspiracies and they are now regularly used in auctions (PORTER, 2005). The development of behavioral screens for cartels outside auctions began only recently. Abrantes-Metz (2012) and Blair and Sokol (2015) provide an overview of the different applications of screens for detecting cartels, and Crede (2016) describes the intuition behind several behavioral cartel screens. In particular, one class of behavioral screens that is directly linked with our research has received much attention recently: variance-based price screens that rely on the idea that the reduced price variance of firms across time or within geographical clusters is an indicator of collusion.

As reported in Harrington (2005), Zitzewitz (2012), economists widely apply this dynamic pricing methodology in an attempt to generate patterns of collusive behavior and, from then on, to formulate and validate a specific hypothesis, to distinguish a competitive pattern from the collusive one. The seminal contribution in this field of research came from Maskin and Tirole (1988). The authors provide a game-theoretic foundation for the classic kinked demand curve equilibrium and Edgeworth cycle. The analysis is based on a model in which firms take turns choosing prices. By using the Markov perfect equilibrium concept, they conclude that a firm's move in any period depends only on the other firm's current price. Using a Markov-switching regression model to estimate both prevalence and structural characteristics of the pricing patterns in retail gasoline markets, Noel (2007) analyzes dynamic pricing in 19 Canadian cities over 574 weeks. The main findings show that sticky-pricing (cycles) is more prevalent when there are few (many) small firms. Wang (2009) studies oligopoly firms' dynamic pricing strategies in the Australian gasoline market before and after the introduction of a unique law that constrains firms to set prices simultaneously and only once per day. The observed pricing behavior, both before and after law implementation, is well captured by the Edgeworth price cycle equilibrium in the Maskin and Tirole dynamic oligopoly model. Thus, the results highlight the importance of price commitment in tacit collusion.

As shown in Lewis (2012), the role of price leadership in coordinating price increases in cycling gasoline retail markets in the U.S[3]. The author concludes that the first price increases tend to stem from retail chains that operate a large number of stations. Following this approach, Clark and Houde (2013) used court documents from a gasoline cartel in Canada to characterize the strategies played by heterogeneous firms to collude and highlight the role of transfers based on adjustment delays during price changes. The cartel leaders systematically allowed the most efficient firms to move last during price-increase episodes to compensate. Atkinson, Eckert and West (2014) did work on another issue related to the retail gasoline pricing in Canada where an event (a refinery fire) seemed to trigger a dramatic change in gasoline price volatility. Using daily retail price data, the authors demonstrated that volatility changes exhibited correspond to an increased frequency of the price cycle, and replacement of the cycle with fixed retail margins. Furthermore, Clark and Houde (2014) uses weekly station-level price data from before and after the cartel's collapse

[3] Lewis and Noel (2011) used a latent regime Markov switching regression framework to show that the constant price movement inherent within the Edgeworth cycle eliminates price frictions and allows firms to pass on cost fluctuations more easily.

to compare pricing behavior in stations affected and unaffected by the investigation. The results indicate that collusion is associated with asymmetric price adjustments and high margins.

In contrast, following the arguments presented in Vasconcelos and Vasconcelos (2008), we use price series rather than margin[4] because there is an ambiguity[5] in the reasons behind the behavior that sustains the cartels. For example, if the profit margin decreases, how can you be sure that this is not a period of punishment after some firm has cheated a cartel agreement? So, the decrease in the profit margin should not be seen merely as an indicator of competition, since firms may be punishing those who deviated. To sum up, on one hand, the cartel can lead to an increase in the profit margin. On the other hand, it may have a punishment phase with lower profits, but this will still be an anti-competitive behavior. We can say that this aspect is a limitation in the methodology of the antitrust authority if the data is restricted in a short period.

Other behavioral issues of economic agents may affect the price variance in the market under analysis. Firms in collusion can practice parallel prices, that is, firms adjust their prices identically and simultaneously, for some common factor[6] of knowledge between them. Such conduct would lead to a similar trajectory of prices among firms, resulting in a low variance[7]. Related to the structural changes in the price series over time, Athey, Bagwell and Sanchirico (2004), Jr and Chen (2006) argue that when firms have a low discount rate on future earnings, collusion equilibrium is given with equal prices. Besides that, when firms exercise some market power, they can also act asymmetrically in the relation between product pricing and cost structure. In this way, the greater the cartel's interference in price formation, the lower the price-to-cost ratio. There is an extensive literature[8] dealing with the problem of asymmetry in collusive markets, with a reasonable consensus on the non-linearity of the relationship between price variations and cost adjustments in collusive markets. Another remarkable feature of collusive markets, according to Perdiguero (2010), refers to coefficients of price variation, which may be relatively different in noncompetitive markets.

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[4] Boroumand et al. (2016) propose a regime-switching model based on mean-reverting and local volatility processes to comprise the market structure of the French fuel retail market. By analyzing the volatility of prices and margins, the authors provided a better understanding of the behavior of oligopolies. In this same market, Porcher and Porcher (2014) found evidence of tacit collusion from the margin analysis, but they emphasize that the collusive behavior in the gasoline market is still an open question.

[5] Theoretically, the stability of the cartel depends on the ability of firms to detect and punish the defectors. To implement the punishment mechanism, cartelized firms can reduce price and, consequently, profit margin. However, this behavior of reduction of the retail price is compatible with that expected in a market environment in which there is an effective competition - without the collusion of the firms.

[6] Such as identical mark-ups, price levels.

[7] We highlight MacLeod (1985), Schmalensee (1987), Rotemberg and Saloner (1990).

[8] Please see Clark and Houde (2014), Silva et al. (2014), Meyer and Cramon-Taubadel (2004).

However, the vast majority of studies cited above use econometric techniques rather than machine learning algorithms and to the best of our knowledge, there are very few papers systematically investigating the performance of the screens based on statistical and computational methods, especially in the gasoline market. In this sense, our research dialogues with the incipient literature on implementing screens to detect bid-rigging cartels (HUBER; IMHOF, 2019). Finally, our framework is also related to studies on screens in markets not characterized by auctions, such as Abrantes-Metz et al. (2006), Jr and Chen (2006), Bolotova, Connor and Miller (2008), Perdiguero (2010), Abrantes-Metz (2012), Atkinson, Eckert and West (2014), Silva et al. (2014).

4.3 GASOLINE CARTELS IN BRAZIL

4.3.1 ANP sample description

We begin this section with a brief description of the Brazilian gasoline market and the database provided by ANP[9]. The available database has continuous weekly price data (Gasoline Station Level) since 1997. With this set of information, it is possible to have a preliminary investigation signaling whether there is any cartel behavior or not in a certain city. The ANP sampling procedures are described as follows. The price collection service, as stated in Pedra et al. (2010), Freitas and Neto (2011), is developed through the structuring and execution of the following steps: (1st) a weekly collection of sales prices to the final consumer and the corresponding acquisition prices by the economic agents selected to integrate the sample defined by the ANP; (2nd) quality control of the information; (3rd) data entry into the system; (4th) creation of a database containing the information specified through contracts; and (5th) forwarding the results to the ANP.

Field planning within each municipality is based on a geographical identification of the resale points within the sample. The weekly collection routes are carried out based on the registration data of resellers in the sample design. The main objective is to optimize the geographical representation. Considering the number of gas stations, a random sample selection is made and collected weekly. The selection procedures must observe the geographic coverage of the municipality as well as guarantee the randomness. Given this sampling plan, it is hard to follow the price dynamics for the same gas station. On the other hand, we have enough information to estimate the city-level statistical moment of the gasoline price distribution.

4.3.2 The cartel cases

Table 1 summarizes the number of cartel and non-cartel observations. The first case we evaluate was set up in the metropolitan region of Belo Horizonte, including the

9 National Agency of Petroleum, Natural Gas and Bio-fuels: <http://www.anp.gov.br.>.

neighboring municipalities of Betim and Contagem. As described in the administrative procedure[10] started in 2014, anonymous complaints date back to the early 2000s, but the hard evidence was collected by the antitrust authority between March 2007 and April 2008. Therefore, we consider the period between January of 2004 and April 2008 as cartel period. To assess the performance of the regulatory agency, we assume the period between January 2014 and April 2019 as the non-cartel period.

Since November 2009, the Brazilian competition authority investigates, monitors and collects information related to the fuel market in Brasilia. During that time, a considerable amount of economic evidence of cartel formation involving distributors and resellers was gathered[11]. In November 2015, CADE decided to enforce a preventive measure in the administrative investigation regarding the gasoline cartel in Brasília. Thus, we consider November 2009 until November 2015 as a cartel period. The non-cartel period runs from December 2015 to April 2019.

In Caxias do Sul[12], the antitrust agency confirmed the evidence that fuel distributors had organized a cartel to fix and standardize prices practiced in fuel resale. The cartel aimed to increase resale margins and eliminate competition, as well as imposing excessive prices. As a result, the municipality's resale margins were much higher than those in other locations in the state. CADE concluded that there was a violation of the economic order and that the gas stations and their managers adopted a uniform and concerted commercial conduct. The cartel was endowed with a high degree of organization, which is why it lasted, at least, between 2004 and 2007, causing immense losses to final consumers. The conviction of the cartel was concluded in 2012. Thus, we consider the period between January 2004 and July 2007 as the cartel period and the period between March 2013 and April 2019 as the non-cartel period.

In São Luís[13], intercepted conversations revealed that the owners of gas stations combined prices and induced other stations that sold the cheaper product to increase their values to strengthen the cartel. Such irregularities would have occurred between January 2010 and October 2014. The investigation also has economic evidence resulting from analyses carried out by the ANP on the São Luís fuel resale market. Frequently, these analyses pointed to the existence of elements that would indicate the possibility of

[10] All information collected is available at <http://en.cade.gov.br/>, in the session Procedure Search. the record of the administrative process related to the Belo Horizonte case is as follows: 08012.007515 / 2000-31.

[11] Administrative Process No. 0800.024581/1994-77 and No. 08012.008859 / 2009-86, available at <http://en.cade.gov.br/> and at <https://tinyurl.com/us8yffd>.

[12] Administrative Process No. 08012.010215 / 2007-96, available at <http://en.cade.gov.br/>.

[13] Administrative Process 08700.002821 / 2014-09 was opened in October 2014, after receipt of transcripts of telephone interceptions duly authorized by the Judiciary of Maranhão, as well as other evidence forwarded to CADE by the Public Ministry of that state that conducted a criminal investigation concerning the same offense. The document is available at <http://en.cade.gov.br/>.

concerted conduct between the gas station owners in the municipality.

Besides, the investigation conducted by the Maranhão Public Prosecutor's Office pointed to a market division among the cartel's participants, to facilitate the operationalization of the illegal agreement, under the coordination of the union. It was also found at the union headquarters a map that divided the city into "corridors", in which the same price for fuel was established.

Table 1 – Number of Cartel and Non-Cartel observations.

	Cartel Obs	Perc. (%)	Non-Cartel Obs	Perc. (%)	Total
Belo Horizonte*	221	45	276	55	497
Brasília	309	63	179	27	488
Caxias do Sul	178	37	306	63	484
São Luís	215	48	236	52	451
Total	923	48.25	997	51.75	1920

Source: Elaborated by the authors.

4.3.3 Statistical screens

Following Huber and Imhof (2019), we consider the statistical screens constructed from the four moments of the distribution of gasoline sales prices in each evaluated city to distinguish between cartel market behavior and non-cartel market behavior.

Standard Deviation & Coefficient of Variation

Price coordination might affect the weekly gasoline sales price dispersion within a city. We thus consider the standard deviation of the gasoline sales price as a screen. Besides, we also consider the coefficient of variation defined as follows as a statistical screen:

$$CV_{c,i} = \frac{s_{c,i}}{\bar{m}_{c,i}}, \tag{4.1}$$

in which the terms $s_{c,i}$ and $\bar{m}_{c,i}$ represents the standard deviation and the mean of the weekly gasoline sales price ($P_{c,i}$), respectively, in a given city c.

Variance

We also consider the variance $\sigma_{c,i}$ of the weekly gasoline sales price within a given city as a screen for detecting cartels. There are theoretical justifications for a variance screen for collusion if it is costly to coordinate price changes or if the cartel must solve an agency problem. There is also some empirical evidence of a decrease in the variance of price during collusion (ABRANTES-METZ et al., 2006).

$$s_{c,i}^2 = \frac{\sum_{i=1}^{n}(P_{c,i} - \bar{m}_{c,i})^2}{n-1}. \tag{4.2}$$

Skewness

Price manipulation may affect the symmetry of the distribution of the weekly gasoline sales price. Thus, for a sample of size n, the the methods of moments estimator of the skewness yields:

$$skew_{c,i} = \frac{m_{3c,i}}{s_{c,i}^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(P_{c,i} - \bar{m}_{c,i})^3}{\left[\frac{1}{n-1}\sum_{i=1}^{n}(P_{c,i} - \bar{m}_{c,i})^2\right]^{3/2}}, \tag{4.3}$$

where $m_{3c,i}$ is the sample third central moment of the weekly retail gasoline price within a given city c.

Kurtosis

Finally, we also investigate whether the cartel affects the "tailedness" of the weekly retail gasoline price distribution through coordination. Thus, we have the following expression for the kurtosis:

$$kurt_{c,i} = \frac{m_{4c,i}}{s_{c,i}^2} - 3 = \frac{\frac{1}{n}\sum_{i=1}^{n}(P_{c,i} - \bar{m}_{c,i})^4}{\left[\frac{1}{n}\sum_{i=1}^{n}(P_{c,i} - \bar{m}_{c,i})^2\right]^{2}} - 3, \tag{4.4}$$

where $m_{4c,i}$ is the fourth sample moment of the sample variance.

4.3.4 Descriptive statistics

Comparing both periods, most screens show fluctuation in the coefficient of variation and standard deviation of prices. The same is observed about variance, skewness, and kurtosis – although in different proportions, in some cities the difference between statistical moments is quite noticeable. Typically, during cartel periods, it is common to see less variance in price distribution. Besides, we assess the expected pattern concerning the other statistical moments on a case-by-case basis, as follows.

Belo Horizonte

Table 2 – Descriptive Statistics - Belo Horizonte*

Screens	Obs	Mean	Std. Dev.	Min	Max
Belo Horizonte*					
Cartel Periods					
Standard deviation	221	0.0748018	0.0161794	0.0414011	0.1166600
Variance	221	0.0058559	0.0025664	0.0017140	0.0136096
Skewness	221	0.7554747	0.7215891	-1.9099710	3.3589700
Kurtosis	221	4.7893610	4.6096630	1.0000000	24.9678600
Coefficient of Variation	221	0.0340085	0.0070236	0.0209430	0.0583500
Non-Cartel Periods					
Standard deviation	276	0.1111709	0.0215818	0.0591717	0.1793580
Variance	276	0.0128231	0.0051184	0.0035013	0.0321694
Skewness	276	0.7070966	0.5847417	-1.9786010	2.5789970
Kurtosis	276	3.8759010	1.6697230	2.0357790	14.3004100
Coefficient of Variation	276	0.0307957	0.0061329	0.0168220	0.0491287

Source: Elaborated by the authors.

Table 2 shows the descriptive statistics for all screens in Belo Horizonte. We can observe that there is a suitable difference both in terms of the means and standard deviation across both groups of periods. The mean of the coefficient of variation is slightly higher in cartel periods. In turn, the mean of the standard deviation screen is approximately 70% lower during the cartel.

Thus, prices are more similar in collusive than in competitive periods. This same intuition fits on the standard deviation of the prices. In this same direction, the variance is 45% lower in cartel periods. As well, on average, both skewness and kurtosis prove to be higher during the cartel period. This pattern leads to a more compressed distribution of prices in cartel periods than in non-cartel periods, suggesting that prices converge when there is a cartel in the retail gasoline market.

Brasília

Table 3 shows the descriptive statistics for the evaluated screens in Brasília. In contrast with the previous case, we can notice that there is a considerable difference both in terms of the means and standard deviation across both groups of periods. As well, the spread of the coefficient of variation is lower in cartel periods with a standard deviation of 0.005, compared to 0.013 for non-cartel periods. The variance reveals to be almost double in non-cartel periods.

This behavior provides shreds of evidence that prices are more similar in cartels.

The price distribution is more asymmetric in cartel periods, and, on average, the kurtosis amounts to 5.0314 in non-cartel periods which more than doubles in non-cartel periods (10.515).

Table 3 – Descriptive Statistics - Brasília.

Screens	Obs	Mean	Std. Dev.	Min	Max
Brasília					
Cartel Periods					
Standard deviation	309	0.0156584	0.0159000	0.0000000	0.0805304
Variance	309	0.0004972	0.0012938	0.0000000	0.0064851
Skewness	309	-0.7774293	2.3707780	-7.8624680	6.8029730
Kurtosis	309	10.5154700	12.1963100	1.0532230	64.6472800
Coefficient of Variation	309	0.0054248	0.0059260	0.0000000	0.0294542
Non-Cartel Periods					
Standard deviation	179	0.1063629	0.0489471	0.0105688	0.2383494
Variance	179	0.0136954	0.0107811	0.0001117	0.0568104
Skewness	179	-0.0893620	1.4663990	-4.0335280	3.7409330
Kurtosis	179	5.0314610	4.2334660	1.4167890	20.8798800
Coefficient of Variation	179	0.0267536	0.0131300	0.0000000	0.0731427

Source: Elaborated by the authors.

Caxias do Sul

Table 4 shows the descriptive statistics for the screens in Caxias do Sul. When compared with Brasília, we observed some similarities concerning the behavior of the coefficient of variation and variance. Note that, during the cartel period, these screens show a much lower variation than that observed in the non-cartel period. These patterns match with the cartel practice.

Besides, the price distribution is more asymmetric in cartel periods. Although the behavior of the retail price of gasoline in Caxias do Sul is not as diverse as that observed in Brasília, it is almost 22% greater during the cartel period.

Table 4 – Descriptive Statistics - Caxias do Sul.

Screens	Obs	Mean	Std. Dev.	Min	Max
Caxias do Sul					
Cartel Periods					
Standard deviation	178	0.0284347	0.0091526	0.0125238	0.0866679
Variance	178	0.0008918	0.0007337	0.0001568	0.0075113
Skewness	178	-0.7019769	1.1253060	-4.4992620	3.3114510
Kurtosis	178	5.1367010	3.3518670	1.3876550	25.0001100
Coefficient of Variation	178	0.0109407	0.0081338	0.0038722	0.0687003
Non-Cartel Periods					
Standard deviation	306	0.0679517	0.0335223	0.0185490	0.2813645
Variance	306	0.0057375	0.0078758	0.0003441	0.0791660
Skewness	306	-0.6077758	1.1112370	-3.2286270	0.0791660
Kurtosis	306	4.2085920	2.2923350	1.3079840	12.0006800
Coefficient of Variation	306	0.0181618	0.0081338	0.0038722	0.0687003

Source: Elaborated by the authors.

São Luís

Table 5 shows the descriptive statistics for the screens in São Luís. We can see that the coefficient of variation and variance is slightly low during the cartel periods. As well, the spread of the skewness is higher in cartel periods with a standard deviation of 1.5057, compared to 1.1089 for non-cartel periods. The mean of the kurtosis amounts to 7.1346 in cartel periods which is almost 60% higher than the non-cartel periods (4.5038).

Finally, in Table 6 we report the Mann-Whitney (MW) and the Kolmogorov-Smirnov (KS) test for the predictors in each city. The Mann-Whitney test can be used to investigate whether two independent samples were selected from populations having the same distribution. In other words, it is used to test the hypothesis of a zero-median difference between two independently sampled populations.

The KS test is a nonparametric test of the equality of one-dimensional probability distributions that can be used to compare two samples. In other words, both tests quantify a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples. The null distribution of these statistics is calculated under the null hypothesis that the sample is drawn from the same distribution.

Accordingly, in Belo Horizonte, the differences observed between detected collusion and no apparent collusion periods are statistically significant at 1% level for the standard deviation, the variance, the kurtosis and the coefficient of variation.

Table 5 – Descriptive Statistics - São Luís.

Screens	Obs	Mean	Std. Dev.	Min	Max
São Luís					
Cartel Periods					
Standard deviation	215	0.0486387	0.0311685	0.0037796	0.1431614
Variance	215	0.0033327	0.0041652	0.0000143	0.0204952
Skewness	215	1.1126350	1.5057160	-3.7490280	4.1108740
Kurtosis	215	7.1346600	4.2591280	1.0000000	24.1474600
Coefficient of Variation	215	0.0189299	0.0126810	0.0013737	0.0599945
Non-Cartel Periods					
Standard deviation	236	0.0726020	0.0255238	0.0107529	0.1388730
Variance	236	0.0059197	0.0040704	0.0001156	0.0192857
Skewness	236	0.2761049	1.1089640	-3.2858770	4.6984360
Kurtosis	236	4.5038870	3.4296160	1.0923390	26.9165300
Coefficient of Variation	236	0.0205283	0.0075034	0.0026962	0.0393634

Source: Elaborated by the authors.

The skewness is significant at 5%. In Brasília, these differences are statistically significant at 1% level for all screens. In Caxias do Sul, the difference between the cartel and non-cartel periods is statistically significant at 1% level for the standard deviation, the variance and the coefficient of variation.

In turn, regarding the KS test, the coefficient of variation is only statistically significant at 5% level. The kurtosis is statistically significant at 5% level for the Mann-Whitney test and 1% level for the KS test. However, the skewness is not statistically significant at 5% level. In São Luís, only the coefficient of variation is not statistically significant at 1% level for the MW test.

Now that we have a better descriptive evaluation of the database, in the following section, we apply the machine learning techniques to train and test models for predicting bid-rigging cartels based on the screens presented in the previous section. Specifically, we consider four different frameworks: Random forest, Lasso and Ridge regressions and Neural Networks (BREIMAN, 1996; BREIMAN, 2001; HO, 1995; MCCULLOCH; PITTS, 1943; RIPLEY, 2007).

Table 6 – Statistical tests for the screens.

Screens	z-statistic	p-value MW	Ksa	p-value KS
Belo Horizonte*				
Standard deviation	15.996	< .0001	0.7022	< .0001
Variance	15.996	< .0001	0.7022	< .0001
Skewness	2.105	0.035	0.2168	< .0001
Kurtosis	4.348	< .0001	0.3012	< .0001
Coefficient of variation	-4.845	< .0001	0.2033	< .0001
Brasília				
Standard deviation	16.994	< .0001	0.8115	< .0001
Variance	16.996	< .0001	0.8117	< .0001
Skewness	4.729	< .0001	0.2684	< .0001
Kurtosis	-6.315	< .0001	0.2857	< .0001
Coefficient of variation	15.653	< .0001	0.797	< .0001
Caxias Do Sul				
Standard deviation	16.656	< .0001	0.8024	< .0001
Variance	16.656	< .0001	0.8024	< .0001
Skewness	0.503	0.6151	0.1013	0.1980
Kurtosis	-3.164	0.0016	0.3012	< .0001
Coefficient of variation	-4.845	< .0001	0.1375	0 .0280
São Luís				
Standard deviation	9.224	< .0001	0.435	< .0001
Variance	9.224	< .0001	0.435	< .0001
Skewness	-6.977	< .0001	0.423	< .0001
Kurtosis	-7.772	< .0001	0.375	< .0001
Coefficient of variation	3.781	0 .0002	0.285	< .0001

Source: Elaborated by the authors.

4.4 MACHINE LEARNING ALGORITHMS

We evaluate the predictions based on several machine learning methods and assess the out of sample performance. To this end, we handle the database as follows. First, we randomize the splitting-testing procedure. In sequence, we divide the database into two subsamples. The training sample estimates the model parameters and contains 75% of the total of observations. The test sample is used for calculating the out of sample predictions and consists of 25% of the observations. After splitting, the presence of a cartel is estimated in the training sample as a function of a range of predictors, namely the original statistical screens.

To assess the performance of out of sample prediction, we consider the following measures: first, the so-called null accuracy, which measures the accuracy that could be achieved by always predicting the most frequent outcome in the database. Second, the so-called score, which measures the proportion of correct classification. Third, miss-classification errors. Fourth, the precision, which measures how often the prediction of cartels is correct. Fifth, the area under the curve (AUC), which measures the relationship between the share of true positive predictions against the share of false-positive predictions at various threshold settings. An area of 1 represents a perfect prediction; an area of 0.5 represents a worthless classifier. To compute the above-mentioned measures, we create a variable that takes the value 1 for predicted cartel probabilities greater than or equal to 0.5 and takes the value 0 otherwise. Then, we compare it to the actual incidence of collusion in the test sample. We repeat random sample splitting into 75% training and 25% test data and all subsequent steps previously mentioned 100 times and take averages of our performance measures over the 100 repetitions.

4.4.1 Random Forest

Random forests are an ensemble learning method largely used in classification tasks that operate by constructing a multitude of decision trees at training time and outputting the class that is the mode of the classes of the individual trees. Moreover, random decision forests correct for decision trees' habit of overfitting (BREIMAN, 2017). Decision trees are a popular method for various machine learning tasks. Random forests are a way of averaging multiple deep decision trees, trained on different parts of the same training set, intending to reduce the variance. This comes at the expense of a small increase in the bias and some loss of interpretability, but generally greatly boosts the performance in the final model.

In our study, we define a vector of features, X, which is composed by the statistical screens − as shown in Table 6 − that will help us to predict the behavior of our target variable, y, that reveals whether a retail gasoline market is under collusion or not. By doing so, the training algorithm for random forests applies the general technique of bootstrap aggregating[14], or bagging, to tree learners. Given a training set $X = x_1, ..., x_n$ with

[14] Bootstrap aggregating (also called bagging) is a machine learning ensemble meta-algorithm designed to improve the stability and accuracy of machine learning algorithms used in

responses $Y = y_1, ..., y_n$, bagging repeatedly (B times) selects a random sample with replacement of the training set and fits trees to the following samples:
For $b = 1, ..., B$:

1. Sample, with replacement, n training examples from X, Y, call these X_b, Y_b;

2. Train a classification tree f_b on X_b, Y_b.

After training, predictions for unseen samples x' can be made by averaging the predictions from all the individual regression trees on x':

$$\hat{f} = \frac{1}{B} \sum_{b=1}^{B} f_b(x'), \tag{4.5}$$

or by taking the majority vote in the case of classification trees. This bootstrapping procedure leads to better model performance because it decreases the variance of the model, without increasing the bias. This means that while the cartel predictions of a single tree are highly sensitive to noise in its training set, the average of many trees is not, as long as the trees are not correlated. Simply training many trees to correctly classify collusive behavior on a single training set would give strongly correlated trees. Additionally, an estimate of the uncertainty of the prediction can be made as to the standard deviation of the predictions from all the individual regression trees on x':

$$\sigma = \sqrt{\frac{\sum_{b=1}^{B}(f_b(x') - \hat{f})^2}{B-1}} \tag{4.6}$$

The number of trees, B, is a free parameter. Typically, a few hundred to several thousand trees are used, depending on the size and nature of the training set. An optimal number of trees B can be found using cross-validation, or by observing the out-of-bag error: the mean prediction error on each training sample x_i, using only the trees that did not have x_i in their bootstrap sample. The training and test error tends to level off after some number of trees have been fit. The above procedure describes the original bagging algorithm for trees. Random forests differ in only one way from this general scheme. It uses a modified tree learning algorithm that selects, at each candidate split in the learning process, a random subset of the features. This process is sometimes called "feature bagging". The reason for doing this is the correlation of the trees in an ordinary bootstrap sample. Thus, if one or a few features are very strong predictors for the cartel, these features will be selected in many of the B trees, causing them to become correlated. An analysis of how bagging and random subspace projection contribute to accuracy gains under different conditions is given by Ho (1995). Typically, for a classification problem with w features, $\sqrt{w}$ features are used in each split. In practice, the best values for these parameters will depend on the problem, and they should be treated as tuning parameters (HASTIE; TIBSHIRANI; FRIEDMAN, 2009). Among the available features, the random

statistical classification. See Breiman (1996) for details.

forest algorithm selects the standard deviation, variance, and coefficient of variation as predictors of cartel behavior in the cities of Belo Horizonte, Brasília and Caxias do Sul. Only in São Luís, the selected features are the standard deviation and the coefficient of variation.

4.4.2 Lasso regression

Lasso estimation was originally formulated for the least-squares models. The so-called lasso regularization corresponds to a penalized logit regression. The penalty parameter improves the prediction accuracy and interpretability of regression models by altering the model fitting process to select only a subset of the provided features for use in the final model rather than using all of them. Besides, it restricts the sum of absolute coefficients on the regressors. Depending on the value of the penalty term, the estimator sets the coefficients of less predictive variables to zero. By doing so, we can select the most relevant features among a possibly large set of predictors. The estimation of the lasso logit coefficients is given by the following optimization problem:

$$\max_{\beta_0,\beta}\left\{\sum_{i=1}^{n}\left[y_i\left(\beta_0+\sum_{k=1}^{w}\beta_j x_{ik}\right)-log\left(1+e^{\beta_0+\sum_{k=1}^{w}\beta_j x_{ik}}\right)\right]-\lambda\sum_{k=1}^{w}|\beta_j|\right\}, \tag{4.7}$$

where β_0, β corresponds to the intercept and slope coefficients on the predictors, respectively. x is the vector of features, i indexes an observation in our database and n is the number of observations. k indexes a predictor and w is the number of features. The parameter $\lambda > 0$ is the penalty term. By cross-validation and randomly splitting the training sample into subsamples, we choose the λ that minimizes the average over the miss-classification error estimates. Most of the subsamples are used to estimate the lasso coefficients under different possible values for λ. One of the subsamples represents the validation database, which we use for predicting cartels based on the different sets of coefficients related to the various penalties and for computing the miss-classification error. After that, we estimate the coefficients of the lasso logit regression by using the training sample. Finally, we predict the cartel probability in the test sample.

4.4.3 Ridge regression

Ridge is a variant of linear regression. It is particularly useful to mitigate the problem of multicollinearity. In general, the method provides improved efficiency in parameter estimation problems in exchange for a tolerable amount of bias. By the ordinary least squares (OLS) we seek to minimize the sum of squared residuals in equation (4.8). Thus, in order to derive a particular solution, we include a regularization term $\lambda\sum_{k=1}^{L}\beta_k^2$ as follows:

$$\hat{\beta}^{ridge}=\min_{\beta_0,\beta}\sum_{i=1}^{n}(y_i-\sum_{k=1}^{L}x_{ik}\beta_k)^2+\lambda\sum_{k=1}^{L}\beta_k^2. \tag{4.8}$$

In other words, Ridge regression adds a squared magnitude on the coefficient β as a penalty term to the loss function. Hence, if $\lambda = 0$, then we have the OLS estimator. On the other hand, if $\lambda \to \infty$, then it will lead to underfitting. In summary, increasing λ decreases the variance and increases the bias of the model. In our study, we use cross-validation to select the value of λ within each evaluated city that minimizes the validation error.

4.4.4 Neural network

Typically, a neural network is composed of an n_l series of layers known as neurons. The layer l of the neural network has M_l neurons in parallel. Each neuron in layer l applies a nonlinear transformation on its M_{l-1} inputs. We can formalize the model as follows:

$$y_k^{(l)} = h^{(l)}\left(\sum_{i=1}^{M_{l-1}} \omega_{ik}^{(l)} y_i^{(l-1)} + \omega_{0k}^{(l)}\right), \quad k = 1, ..., n_l, \tag{4.9}$$

where $a_k^{(l)} = \sum_{i=1}^{M_{l-1}} \omega_{ik}^{(l)} y_i^{(l-1)}$ is the activation of the neuron k and the term $\omega_{0k}^{(l)}$ measures the bias associated to an entry $y_0^{(l-1)} = 1$. The term $h^{(l)}$ is the activation function of the neurons in layer l. By definition, we have that $y_i^0 = x_i$ where $i = 1, ...M_0$ represents the inputs of the neural network. Regarding the target variable, we have that $y_i^{nl} = y_i^0$, in which $i = 1, ...M_{nl}$ represents the output of the neural network. Thus, the neural network has $M_{nl} = M_0$ outputs. In our study, the inputs of the neural network are the statistical moments of the retail gasoline price. The output is our so-called target variable, i.e., the cartel predictions.

4.5 Empirical results

We start our empirical analysis by presenting the results through the confusion matrix for each machine learning techniques[15]. In predictive analytics, a confusion matrix is a table with two rows and two columns that reports the number of false-positives, false-negatives, true positives, and true negatives. In statistical hypothesis testing, a false-positive (negative) corresponds to the Type I (II) error. Thus, each row of the matrix represents the instances in a predicted class (cartel and non-cartel periods) while each column represents the instances in an actual class. This allows a more detailed analysis than mere proportion of correct classifications (score). A score is not a sufficient metric for the real performance of a classifier. As it does not tell us the underlying distribution of response values, it will yield misleading results if the data set is unbalanced (FAWCETT, 2006; SAMMUT; WEBB, 2011; POWERS, 2011). In other words, it does not inform about the types of errors the classifier is making. For example, if there were 95 cartel observations and only 5 non-cartel observations in the data, a particular classifier might

15 We repeat the classification procedure 100 times and the values are based on the average of each of the metrics computed from the confusion matrix.

classify all the observations as cartels. The overall score would be 95%, but in more detail, the classifier would have a 100% sensitivity, i.e., the recognition rate for the cartel class but a 0% recognition rate for the non-cartel class.

Belo Horizonte

As the confusion matrix in Belo Horizonte reveals, by adding all the entries for each machine learning algorithms as shown in Table (7), we evaluate the average of the predictions based on 125 observations. Before going deep into this analysis, it is important to highlight the following point: by convention, we describe the class encoded as 1 as the positive class (cartel) and the class encoded as 0 as the negative class (non-cartel). In that sense, the true positive (negative) represents the case in which the model correctly predicted a 1 (0) value. As well, we consider a classification threshold $\iota = 0.5$.

Table 7 – Confusion Matrix considering $\iota = 0.5$ - Belo Horizonte.

Random Forest

		Predicted	
		Non-cartel (0)	cartel (1)
Actual	Non-cartel (0)	66	1
	cartel (1)	0	58

Lasso Regression

		Predicted	
		Non-cartel (0)	cartel (1)
Actual	Non-cartel (0)	63	4
	cartel (1)	12	46

Neural Networks

		Predicted	
		Non-cartel (0)	cartel (1)
Actual	Non-cartel (0)	65	2
	cartel (1)	5	53

Ridge Regression

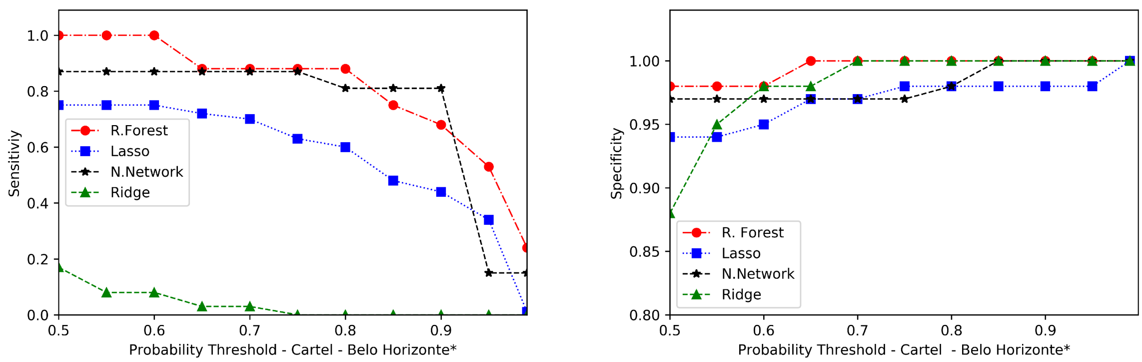
		Predicted	
		Non-cartel (0)	cartel (1)
Actual	Non-cartel (0)	59	8
	cartel (1)	48	10

Source: Elaborated by the authors.

By looking at the Random Forest predictions, we see on the bottom right the number of true positives, which indicates that in 58 cases the classifier correctly predicted the cartel period. On the upper left, we observe the number of true negatives, which indicates that in 66 cases the classifier correctly predicted the non-cartel periods. On the upper right, we have the number of false-positives. Note that it indicates that only in 1 case the classifier incurred a Type I error. However, on the bottom left we see that the random forest does not incur a Type II error.

To compute the classification accuracy score, we first must add true-positives and true-negatives. In sequence, we divide that amount by the total number of observations, i.e., the score is equal to $(66+58)/125 = 99.2\%$. When comparing the random forest score with the results derived from the confusion matrices of the other algorithms, we see that the closest score is that of the neural network. The model with the lowest classification score was the ridge regression, where $(59+10)/125 = 55.2\%$. As well, we can assess the classification error metric by adding the false-positives and false-negatives and dividing that amount by the total number of observations. In that sense, we can infer that the random forest misclassification error is given by $(0+1)/125 = 0.8\%$. This is the smallest classification error observed for Belo Horizonte. In contrast, for the ridge regression we have an error given by $(48+8)/125 = 44.8\%$.

Figure 17 – Sensitivity and Specificity by restricting ι - Belo Horizonte.



(a) (b)

Source: Elaborated by the authors.

Figure 17 reports two metrics used for evaluating the trade-offs in classification accuracy. The sensitivity assesses the true positive rate and aims to measure the proportion of actual positives correctly identified. The specificity is also known as the true negative rate and measures the proportion of actual negatives correctly identified. For both sensitivity and specificity, the best possible value is 1. In the confusion matrix, sensitivity is calculated by dividing the true positives by the total of the bottom row. For the random forest classifier, we have that the sensitivity is equal to $58/(0+58) = 100\%$. In contrast, note that the ridge regression classifier has the lowest true positive rate. Specificity is calculated by dividing the true negatives by the total amount in the top row.

Hence, for the random forest classifier, the Specificity measure is given by $66/(66+1) = 98.5\%$. The Ridge regression has the lowest true negative values $59/(59+8) = 88\%$. Finally, from the confusion matrix, we can calculate the precision metrics by dividing true positives by the total of the right column. By doing so, the Random Forest classifier has a precision equals to $58/(58+1) = 98.3\%$. The performance of the Neural Network is

$53/(53+2) = 96.3\%$. Lasso and Ridge show reasonable precision rates, but relatively smaller than the others.

As expected, through Figures 17a and 17b, the correct classification rate in non-cartel periods increases in the probability threshold for the decision rule (false-positive results decrease). In contrast, the correct classification rate in cartel periods deteriorates much faster in the threshold (false-negative results increase). In other words, the antitrust agency would be able to minimize the false-positive rates (1− specificity) by increasing the decision rule threshold to a value closer to 0.7.

In this scenario, the performance of the Random Forest and Ridge Regression predictors allows for minimal risk of false-positives outcomes. As well, for the Ridge algorithm, we must observe that the 0.7 classification rule, leads to a false-negative rate (1− sensitivity) closer to 1. In contrast, the Neural Network and the Random Forest classifiers show approximately 15% of false-negative outcomes. In summary, the gain of reducing the risk of false-positives, therefore, induces a disproportionate increase in false-negatives.

Moreover, any further tightening of the decision rule would lead to an even more severe increase of false-negatives. At a probability threshold of 0.8, Random Forest shows the best performance. It, therefore, seems that for the gasoline cartel in Belo Horizonte, the best-suited probability threshold lies between values of 0.5 and 0.7. One advantage of combining screening methods and machine learning consists of quantifying the trade-off regarding false-positives and false-negatives so that the regulators are capable to determine the decision rule that optimally matches their needs.

We conclude the performance of our binary classifiers by assessing the area under the curve (AUC) metrics. It provides useful information regarding how well the classifiers are separating the cartel periods from the non-cartel periods. In general, the AUC represents the probability that a classifier will rank a randomly chosen positive observation higher than a randomly chosen negative observation. Thus, the closer the AUC is to 1, the better the classifier. As Table 11 reveals, the Random Forest predictor has the greater AUC.

Brasília

Differently from the previous case, Table 8 reveals that the Lasso Regression shows the best score index $(44+73)/122 = 95.9\%$. Besides, it presents a classification error equals to $(3+2)/122 = 4.1\%$. The Random Forest algorithm also shows a reasonable performance. In terms of sensitivity and specificity, when considering a classification threshold equals to 0.5, the Lasso Regression classifier shows the best prediction outcomes.

The true positive and true negative rates are given by $73/(2+73) = 97.3\%$ and

$44/(44+3) = 93.6\%$. The precision index of the Lasso Regression is slightly higher $73/(73+6) = 96.1\%$ than the Random Forest.

Table 8 – Confusion Matrix considering $\iota = 0.5$ - Brasília.

Random Forest

		Predicted Non-cartel (0)	Predicted cartel (1)
Actual	Non-cartel (0)	44	3
	cartel (1)	3	72

Lasso Regression

		Predicted Non-cartel (0)	Predicted cartel (1)
Actual	Non-cartel (0)	44	3
	cartel (1)	2	73

Neural Networks

		Predicted Non-cartel (0)	Predicted cartel (1)
Actual	Non-cartel (0)	40	7
	cartel (1)	4	71

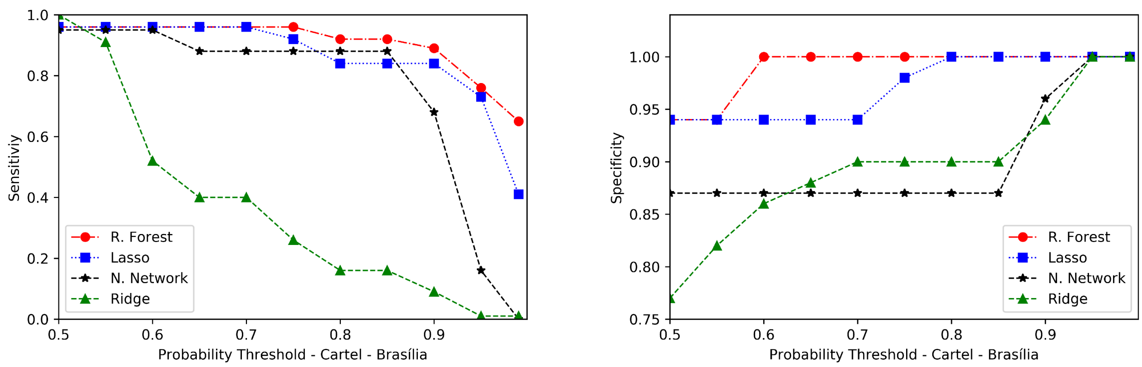
Logistic Regression

		Predicted Non-cartel (0)	Predicted cartel (1)
Actual	Non-cartel (0)	33	16
	cartel (1)	0	73

Source: Elaborated by the authors.

Figure 18 summarizes the trade-offs in classification accuracy for the gasoline cartel in Brasília. As expected, through Figures 18a and 18b, the observe that false-positive results decreases in the probability threshold. However, the false-negative rate in cartel periods increases much faster in the threshold. To minimize the false-positive rates (1−specificity), the optimum decision rule threshold should be greater than 0.6.

Figure 18 – Sensitivity and Specificity by restricting ι - Brasília.



(a) (b)

Source: Elaborated by the authors.

In this case, the performance of the Random Forest minimizes the false-positive rate. Ridge Regression predictors allow for minimal risk of false-positives outcomes. This same condition is true for Lasso Regression when the threshold is greater than 0.8. Yet, we must observe that a classification rule greater than 0.75, leads to a false-negative rate (1− sensitivity) closer to 10% for the Random Forest. The Lasso predictors show approximately 15% of false-negative outcomes. As before, the benefits of reducing the risk of false-positives is unreasonable for the increase in false-negatives. Therefore, at a probability threshold of 0.5, Lasso Regression shows the best performance.

When we increase the decision rule by considering a threshold greater than 0.75, Random Forest proves to be the best algorithm for classifying the gasoline cartel in Brasília. Judging by the AUC criterion, both predictors have a satisfactory classification rate, but the Ridge predictor shows the best performance in this regard (AUC = 88.3%). On the other hand, taking into account all the evaluation metrics, from Table 11, we can conclude that LASSO regression, on average, performs subtly better than Random Forest.

Caxias do Sul

The confusion matrix in Table 9 shows that the Random Forest provides the best score index $(75 + 40)/121 = 95\%$. The classification error is given by equals to $(2 + 4)/121 = 5\%$. Considering a classification threshold equal or greater than 0.5, the Random Forest shows the best prediction outcomes. For a probability decision rule equals 0.5, the true positive and true negative rates are given by $40/(4 + 40) = 90.9\%$ and $75/(75 + 2) = 97.4\%$. The precision index of the Ridge Regression model is the largest $4/(0 + 4) = 100\%$.

Figure 19 illustrates how sensitivity and specificity react to an increase in the probability threshold. By comparing the outcomes represented in Figures 19a and 19b, we see that the false-negative rate (1− sensitivity) is closer to 100% for the Ridge algorithm. The Random Forrest predictors show approximately 15% of false-negative outcomes for a threshold probability equal or lower than 0.65.

When we narrow the decision rule, especially assuming values greater than 0.75 we affect both the sensitivity and the specificity of the classification algorithms. Note that for the antitrust authority, it is not interesting to adopt the Ridge model to identify the cartel in Caxias do Sul. Note that for the antitrust authority, it is not interesting to adopt the Ridge model to identify the cartel in Caxias do Sul. In other words, a high specificity rate is not a sufficient condition to minimize classification errors.

Table 9 – Confusion Matrix considering $\iota = 0.5$ - Caxias do Sul.

Random Forest			
		Predicted	
		Non-cartel (0)	cartel (1)
Actual	Non-cartel (0)	75	2
	cartel (1)	4	40

Lasso Regression			
		Predicted	
		Non-cartel (0)	cartel (1)
Actual	Non-cartel (0)	73	4
	cartel (1)	7	37

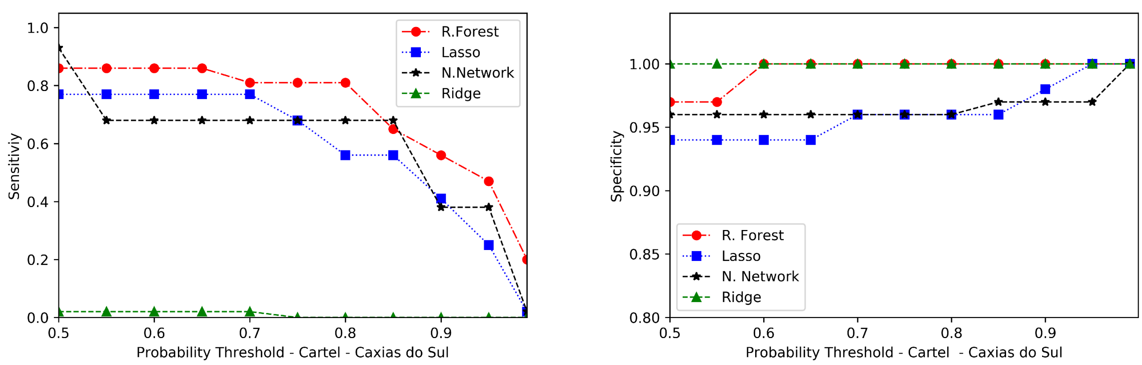
Neural Networks			
		Predicted	
		Non-cartel (0)	cartel (1)
Actual	Non-cartel (0)	73	4
	cartel (1)	3	41

Logistic Regression			
		Predicted	
		Non-cartel (0)	cartel (1)
Actual	Non-cartel (0)	77	0
	cartel (1)	40	4

Source: Elaborated by the authors.

To demonstrate this, we assess the (poor) performance of the Ridge model incorrectly classifying observations as a cartel period (sensitivity). Thus, the classifier that best responds to the data - indicating how many observations were correctly identified as a cartel period (sensitivity) and how many observations were correctly identified as a non-cartel period (specificity) - is the Random Forest.

Figure 19 – Sensitivity and Specificity by restricting ι - Caxias do Sul.



(a) (b)

Source: Elaborated by the authors.

In a complementary way, we see that by the AUC criterion as in Table 11, we also conclude that, on average, the Random Forest estimators show the best performance in predicting the gasoline cartel in Caxias do Sul. It is also worth noting that, on average, the neural network performed better than the LASSO and Ridge regressions.

São Luís

The models that best classify the cartel in São Luís are Random Forest and Neural Networks, respectively. Using the confusion matrix available in Table 10, we see that the power of Lasso regression, whose classification error $(14+7)/113$ is approximately 18.6%, is almost three times greater than the error calculated for Random Forest $(3+3)/113 = 5.03\%$. For Neural Network this measure is equal to $(7+3)/113 = 8.9\%$. Ridge Regression presents a classification error given by $(11+6)/113 = 15.1\%$.

Table 10 – Confusion Matrix considering $\iota = 0.5$ - São Luís.

Random Forest

		Predicted Non-cartel (0)	Predicted cartel (1)
Actual	Non-cartel (0)	61	3
	cartel (1)	3	46

Lasso Regression

		Predicted Non-cartel (0)	Predicted cartel (1)
Actual	Non-cartel (0)	57	7
	cartel (1)	14	35

Neural Networks

		Predicted Non-cartel (0)	Predicted cartel (1)
Actual	Non-cartel (0)	61	3
	cartel (1)	7	42

Ridge Regression

		Predicted Non-cartel (0)	Predicted cartel (1)
Actual	Non-cartel (0)	58	6
	cartel (1)	11	38

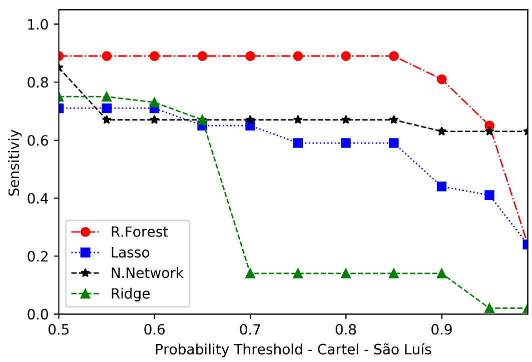
Source: Elaborated by the authors.

Regarding the precision measure, the Random Forest shows the best performance in relation to the true positives cartel observations $46/(46+3) = 93.87\%$. The model that comes closest to this rate is the Neural Network $(42/42+3) = 93.3$. The proximity between the quality of the predictions of both models remains for all the other statistics. Thus, considering the probability threshold equals to 0.5 and judging by the set of measures, on average, we observe from Figure 20 that Random Forest is more accurate.

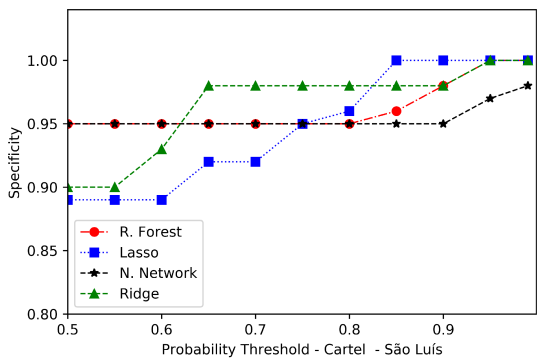
For a decision rule less than or equal to 0.95, as reported in Figure 20a, the Random Forest algorithm shows the lowest rate for false-negative outcomes. Withal, there is a scenario in which the sensitivity of Neural Network is equivalent to that of Random Forest.

Regarding specificity, for a threshold between 0.6 and 0.8, the Ridge regression presents better performance, being surpassed by Lasso Regression for intervals between 0.8 and 0.95. At this point, by Figure 20b, we see that the Ridge, Lasso, and Random Forest models are equivalent regarding the proportion of actual negatives that are correctly identified. Only the Neural Network has a slightly lower performance.

Figure 20 – Sensitivity and Specificity by restricting ι - São Luís.



(a)



(b)

Source: Elaborated by the authors.

4.6 ANTITRUST AUTHORITIES & COMPETITION POLICY

An important question for the attractiveness of our method is whether it produces robust results that allow it to be extrapolated to other sectors of economic activity. The statistical screen approach is expected to have a good performance, even in other sectors or countries, where price dynamics may vary from those considered in this proposal.

Besides, the flexibility of this approach is facilitated by the use of several distinct cartel screens that are sensitive to different characteristics of markets and therefore potentially cover different price collusion mechanisms. Furthermore, in contrast with other detection methods that require data on cost variables, such as the econometric approach proposed by Bajari and Ye (2003), in our approach firm-level cost data and daily price data are not necessary conditions for the antitrust agency to detect collusion.

In the case of cartels, one of the most challenging objectives for regulatory agencies is the legal and economic consistency of the prosecution. In this sense, price distribution screen-based may overcome some drawbacks of the traditional econometric approach (HUBER; IMHOF, 2019).

In other words, simple screens are not as time-consuming as the structural econometric methods which demand non-observable variables such as costs and produce many false-negative results when applied in real cases (BAJARI; YE, 2003). As well, classification errors generate a very high opportunity cost for the regulator, in addition to damaging the reputation of the Antitrust authority (ABRANTES-METZ, 2012).

Table 11 – Performance of the machine learning algorithms.

	Null Accuracy (%)	Score (%)	Error (%)	Precision (%)	AUC (%)
Belo Horizonte*					
Random Forest	46.40	99.20	0.80	98.30	99.90
LASSO Regression	46.40	87.20	12.80	92.00	94.30
Neural Networks	46.40	94.40	5.60	96.30	89.90
Ridge Regression	46.40	55.20	44.80	55.50	70.80
Brasília					
Random Forest	61.50	95.10	4.90	96.00	86.80
LASSO Regression	61.50	95.90	4.10	96.00	86.90
Neural Networks	61.50	90.90	8.20	91.00	85.90
Ridge Regression	61.50	86.90	13.10	82.00	88.30
Caxias do Sul					
Random Forest	36.30	95.00	5.00	95.20	98.40
LASSO Regression	36.30	90.90	9.10	90.20	94.50
Neural Networks	36.30	94.20	5.80	91.10	96.20
Ridge Regression	36.30	66.90	33.10	100.00	73.80
São Luís					
Random Forest	43.30	94.60	5.40	93.80	98.60
LASSO Regression	43.30	81.40	18.60	83.30	90.10
Neural Networks	43.30	91.10	8.90	93.30	96.60
Ridge Regression	43.30	84.90	15.10	86.40	91.00

Source: Elaborated by the authors.

Given the above, it is important to reinforce the attractiveness of our method to raise the quality of the decisions made by the policymaker. Given the adaptability of the machine learning algorithms, in addition to the traditional variance screen, we consider the third and fourth moments of the distribution of the gasoline retail price to map market behaviors that are harmful to competition and consumer welfare.

Imhof, Karagök and Rutz (2018) proposed a similar discussion and Huber and Imhof (2018) investigates bid-rigging cartels in the Swiss civil construction sector. In this way, our assessment of the Brazilian gasoline retail market contributes to the regulator, once we demonstrate how the predictions of the statistical screens combined with machine learning are robust and can be generalized to different markets in different countries.

Concerning our case study, we recommend the use of our screens by adopting some practices, as follows. First, in most of the cases, the coefficient of variation and the standard deviation reveals to be the most powerful features. In this way, they help us to infer about the negative relationship between the variance of the retail price of gasoline and the cartel probability. Therefore, low price variance suggests a higher probability of a cartel (ABRANTES-METZ et al., 2006). On the other hand, in some contexts, both skewness (asymmetry) and kurtosis reveal to be relevant in the correct prediction of cartel probability. Thus, we can see the relevance of all statistical moments.

Ultimately, we have a range of predictors that can act in a complementary and substitute manner with each other, increasing the quality of the economic evidence on the formation of a cartel. Finally, regarding the trade-offs in reducing false-positive *vs.* false-negative outcomes, an appropriate strategy would be to increase the probability threshold between 0.6 and 0.75. This practice might reduce incorrect predictions among truly non-cartel periods (false-positives) at the expense of increasing the number of truly cartel periods (false-negatives).

4.7 CONCLUSION

In this paper, we integrated many different machine learning techniques with statistical screens based on the moments of the distribution of the gasoline retail price to correctly identify and predict cartel market behavior. On this matter, we evaluated the out of sample performance of four different models: Random Forest, Lasso, Ridge and Neural Networks. By splitting the data into testing and training samples, we estimate the models for four Brazilian cities already judged and condemned by cartel: Belo Horizonte, Brasília, Caxias do Sul and São Luís.

Considering an average of the overall accuracy, the models correctly predicted around 87% of the cartel periods. In a comparison between the four models, we highlight the predictive efficiency of the models according to the following ranking: Random Forest, Lasso Regression, Neural Network, and Ridge Regression. Considering all cities, the Random Forrest algorithm, on average, showed a score of 95% correct classifications – for both cartel and non-cartel periods.

Even when increasing the probability threshold, Random Forest was the most stable model, preserving high levels of sensitivity and specificity. We also found evidence that both asymmetry and kurtosis are features that increases the algorithms' performance. These features work in a complementary way or even replacing the variance and the coefficient of variation in the cartel prediction. Thus, we empirically reinforce the intuition that relying on strong assumptions regarding the structural relationship between a given screen and the probability of cartel does not assure high predictive power.

This, in some way, confirms the flexibility and the generalization of our approach - which is based on the simple hypothesis that if the formation of a cartel in the retail of gasoline affects the statistical moments of the price distribution, we can observe these changes in pattern through time-saving machine learning techniques.

Finally, we offer possible recommendations to the regulator regarding best practices for using our approach. In this way, we reinforce the benefits that the simplicity of the model in terms of data and implementation can offer. In contrast, we emphasize the costs and damage to the reputation of the antitrust authority, inherent in the trade-off between reducing false-positives *vs.* false-negatives.

Among the possible extensions of this paper, we suggest the incorporation of spatial elements and regional characteristics regarding the gasoline retail market in the analysis. Promising research would be to establish an approximation between the Edgeworth price cycle approach, passthrough of upstream cost shocks, response asymmetry and variance screens as discussed in Eckert (2013), with machine learning algorithms. Another relevant contribution would be to evaluate the performance of statistical screens in other countries' gasoline market, whose price dynamics might differ from those found in Brazil.

5 CONCLUSIONS

In this thesis, we studied the circumstances in which cartel agreements proved to be stable. Our study may provide some practical competition policy prescriptions to guide the antitrust authority regarding cartel deterrence. Considering a dynamic competition, the first essay distinguishes the collusive behavior in regional and digital markets. Introducing an altruistic punishment behavior in legal cartel agreements, we found some pieces of evidence that collusion is more stable in a digital economy environment. In regional markets, cartel stability is increasing in the number of high-efficiency firms. On the other hand, while high-efficiency firms collude with one another, low-efficiency firms behave like free-riders, taking advantage of the competitive fringe. The antitrust authority must be attentive to these insights to optimize its performance to inhibit regional and digital cartel agreements.

With this in mind, the second essay sheds light on the stability of so-called illegal cartels. Thus, retaliation mechanisms are not standard. Given that cartels are criminal organizations, our results showed a pattern of stability different from that obtained in the first essay. The asymmetry between firms increases the stability of the collusion and, the greater the number of high-cost firms, the more stable the cartel. Furthermore, the insights provided by the different strands of game theory, as presented in the first and second essays, respectively, are complementary.

According to our theoretical contributions, the antitrust authority must be aware that the likelihood of cartel formation may be increasing in regions where there is a greater asymmetry between firms. In other words, taking into account the cities evaluated in our empirical study of the Brazilian gasoline market, the regulator should give greater focus to regions where there is a higher proportion of low-efficiency firms, since the characteristics of the market may favor the anti-competitive practice. Besides, we emphasize the need for careful action by the antitrust authority to deter illegal cartels. On this matter, the policymaker, being aware that the cartel behaves like a criminal organization, can act jointly with other law enforcement agencies.

Finally, our empirical approach via machine learning algorithms can be thought of as a data mining platform for the improvement of behavioral screens, which would reduce the financial effort of detecting cartels relying exclusively on leniency programs. In this regard, we offer an even more technical policy prescription on the deterrence of cartel agreements. As well, the ability of machine learning algorithms to recognize behavioral patterns of price dynamics are useful in other sectors of the economy.

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APPENDIX A – ALGEBRAIC DERIVATIONS

Algebraic Derivation for the Setting with 3 Homogeneous Firms

The Oligopoly Payoff with 3 Honest Firms

The F.O.C. derived from equation (3.2) yield the following expression after adding and subtracting $\delta_i p_{j,i}$:

$$p_{j,i}(6+5\delta_i) = 3b_i + \delta_i P_i + (3+2\delta_i)k, \qquad j = 1,2,3. \tag{A.1}$$

Equation (A.1) reveals that $p_{1,i} = p_{2,i} = p_{3,i}$. Let $p_{j,i} = p_i, j = 1,2,3.$. Now, note that $P_i = 3p_i$. Substituting this into (A.1) and solving the implied expression, we obtain:

$$p_i^O = \frac{3b_i + (3+2\delta_i)k}{2(3+\delta_i)}. \tag{A.2}$$

Combining equations (3.1) and (A.2), we have:

$$q_i^O = \frac{(b_i - k)(3+2\delta_i)}{2(3+\delta_i)} \qquad j = 1,2,3. \tag{A.3}$$

Combining equations (3.2), (A.2) and (A.3) yields the payoff in equation (3.3).

The Cartel Payoff with 3 Dishonest Firms

From equation (3.4), we have:

$$\sum_{F=1}^{3} E\pi_{F,i}^M = (p_{1,i}+p_{2,i}+p_{3,i})b_i + \frac{\delta_i}{3}[p_{1,i}(p_{2,i}+p_{3,i}) + p_{2,i}(p_{1,i}+p_{3,i}) + p_{3,i}(p_{1,i}+p_{2,i})]$$
$$- \frac{(3+2\delta_i)}{3}(p_{1,i}^2 + p_{2,i}^2 + p_{3,i}^2) - \left\{3b_i k + \frac{\delta_i}{3}[2(p_{1,i}+p_{2,i}+p_{3,i})k]\right.$$
$$\left. - \frac{(3+2\delta_i)}{3}(p_{1,i}+p_{2,i}+p_{3,i})k\right\} - \sigma_i f.$$

After opening the summation and maximizing (3.4) with respect to prices yields the following conditions for firm 1:

$$\frac{\partial \pi_{1,i}}{\partial p_{1,i}} = 0 \rightarrow b_i + \frac{2\delta_i}{3}(p_{2,i}+p_{3,i}) - \frac{2(3+2\delta_i)}{3}p_{1,i} + k = 0.$$

Adding and subtracting $2\delta_i p_{1,i}$:

$$6p_{1,i}(1+\delta_i) = 3b_i + 2\delta_i P_1 + k.$$

Generalizing for j homogeneous firms, we have:

$$6p_{j,i}(1+\delta_i) = 3b_i + 2\delta_i P_i + k. \tag{A.4}$$

Equation (A.4) implies that $p_{1,i} = p_{2,i} = p_{3,i}$. Let $p_{j,i} = p_i^M, j = 1, 2, 3$. As $P_i = 3p_i^M$, we obtain the following result:

$$p_i^M = \frac{b_i}{2} + k. \tag{A.5}$$

Combining equations (3.1) and (A.5) yields:

$$q_{j,i} = q_i^M = \frac{b_i}{2} - k \qquad j = 1, 2, 3. \tag{A.6}$$

Equations (A.5) and (A.6) imply that each firm earn the expected payoff given by equation (3.5) in each period.

Deviation and Retaliation

The first order condition derived from equation (3.6) yields:

$$p_{3,i} = \frac{3 + \delta_i}{2(3 + 2\delta_i)} b_i + \frac{3 + 4\delta_i}{2(3 + 2\delta_i)} k. \tag{A.7}$$

As the payoff (3.6) reveals,

$$q_{3,i} = \frac{1}{3}[3b_i + (b_i + 2k)\delta_i - p_{3,i}(3 + 2\delta_i)]. \tag{A.8}$$

Combining equations (A.7) and (A.8) yields:

$$q_{3,i} = \frac{3 + \delta_i}{6} b_i - \frac{k}{2}. \tag{A.9}$$

Hence, firm 3's expected payoff in the first period is given by equation (3.7). Since the quantity demanded from each cartel member equals

$$q_i^M = \frac{1}{3}[3b_i + (p_i^M + p_{3,i})\delta_i - p_i^M(3 + 2\delta_i)] = \frac{1}{3}[3b_i + p_{3,i}\delta_i - p_i^M(3 + \delta_i)], \tag{A.10}$$

combining equations (A.7) and (A.10) yields:

$$q_i^M = \frac{[9 + \delta_i(6 - \delta_i)]b_i - 3(6 + 5\delta_i)k}{6(3 + 2\delta_i)}. \tag{A.11}$$

Given (A.11), each cartel member earns the expected payoff given by equation (3.8). In the second period, the F.O.C derived from equation (3.9) yields:

$$p_{3,i} = \frac{3b_i + \delta_i P_{-3,i} + (3 + 2\delta_i)k - s}{2(3 + 2\delta_i)}. \tag{A.12}$$

Given (A.12), the quantity sold by firm 3 in the retaliation period is

$$q_{3,i} = \frac{3b_i + \delta_i P_{-3,i} - (3 + 2\delta_i)k - s}{6}. \tag{A.13}$$

The constraint in equation (3.10) follows from (A.13). By assuming that the inequality in (3.11) holds. Then, $p_{3,i} = q_{3,i} = 0$ and the F.O.C. derived from equation (3.13) yield:

$$p_{1,i} = p_{2,i} = p_i^M = \frac{3b_i - 2\delta_i c}{4} + \frac{k}{2}. \tag{A.14}$$

Given (A.14), we have:

$$q_{1,i} = q_{2,i} = q_i^M = \frac{3b_i}{2} + \delta_i c - k. \tag{A.15}$$

Thus, we can find equations (3.14). Combining the inequality in (3.11) with equation (A.14) yields the necessary condition for the cartel to steam fir 3's product as demonstrated in equation and (3.15). When the inequality in (3.12) holds, the F.O.C derived from equation (3.18) yields:

$$p_{1,i} = p_{2,i} = p_i^M = \frac{3(6+5\delta_i)+\delta_i^2}{4(3+2\delta_i)(3+\delta_i)} b_i + \frac{6+5\delta_i}{4(3+2\delta_i)} k. \tag{A.16}$$

Given (A.16), we obtain:

$$q_{1,i} = q_{2,i} = q_i^M = \frac{[3(6+5\delta_i)+\delta_i^2]b_i - 3[6+\delta_i(5-\delta_i)]k}{4(3+2\delta_i)}. \tag{A.17}$$

Equations (A.16) and (A.17) provide the expected payoff earned by each cartel member as in expression (3.19). Combining equations (A.12) and (A.16) yields:

$$p_{3,i} = \frac{[9(6+8\delta_i+3\delta_i^2)+\delta_i^3]b_i + [54(1+2\delta_i)+13\delta_i^2(3+\delta_i)]k}{4(3+2\delta_i)^2(3+\delta_i)}. \tag{A.18}$$

Combining equations (A.13) and (A.16), we have:

$$q_{3,i} = \frac{[9(6+8\delta_i+3\delta_i^2)+\delta_i^3]b_i - [9(6+8\delta_i+3\delta_i^2)+3\delta_i^3]k}{4(3+2\delta_i)^2(3+\delta_i)}. \tag{A.19}$$

Given equations (A.18) and (A.19), we obtain firm 3's expected payoff as in expression (3.20).

N Homogeneous Firms

The F.O.C derived from equation (3.28) yields:

$$p_{-j,i} = \frac{N(2+\delta_i)-\delta_i}{4[N(1+\delta_i)-\delta_i]} b_i + \frac{N(1+2\delta_i)-2\delta_i}{2[N(1+\delta_i)-\delta_i]} k. \tag{A.20}$$

As the payoff (3.28) reveals,

$$q_{-j,i} = \frac{Nb_i + (N-1)p_i^M \delta_i - p_{-j,i}[N+(N-1)\delta_i]}{N}. \tag{A.21}$$

Combining equations (A.20) and (A.21) yields:

$$q_{-j,i} = \frac{N(2+\delta_i)-\delta_i}{4} b_i - \frac{1}{2} Nk. \tag{A.22}$$

Hence, we can calculate firm $n_{-j,i}$'s expected payoff in the first period as in expression (3.29). Since the quantity demand from each $N-1$ cartel members equals:

$$q_i^M = \frac{Nb_i + \left[(N-2)\,p_i^M + p_{-j,i}\right]\delta_i - p_i^M\,[N+(N-1)\,\delta_i]}{N}, \tag{A.23}$$

combining equations (A.21) and (A.23) yields:

$$q_i^M = \frac{1}{4}\frac{2\,(1+\delta_i)\,(b_i-2\,k)\,N^2-\delta_i\,(b_i\delta_i+2\,b_i-2\,k)\,N+b_i{\delta_i}^2}{N\,(N\,(1+\delta_i)-\delta_i)}. \tag{A.24}$$

Given (A.24), we can compute each $N-1$ cartel member expected payoff in the first period as in equation (3.30).

Algebraic Derivation for the Setting with Heterogeneous Firms

Oligopoly Price for the Low-cost Firm

As before, we start the analysis considering the polar case in which all the 3 firms are honest. Taking the other firms' price $(p_{2,i}, p_{3,i})$ choice as given and considering $k_{1,i}=0$, the oligopoly profit maximization for the low-cost firm is given as follows:

$$\pi_{1,i}^O = \frac{1}{3}p_{1,i}[3b_i+\delta_i P_{-1,i}-p_{1,i}(3+2\delta_i)].$$

The fist order condition yield:

$$\frac{\partial\pi_{1,i}^O}{\partial p_{1,i}} = 0 \rightarrow b_i+\frac{\delta_i}{3}(p_{2,i}+p_{3,i})-\frac{2(3+2\delta_i)}{3}p_{1,i}=0.$$

$$3b_i+\delta_i P_{-1,i}=2p_{1,i}(3+2\delta_i).$$

Adding and subtracting $\delta_i p_{1,i}$:

$$3b_i+\delta_i P_i = p_{1,i}(6+5\delta_i). \tag{A.25}$$

Setting $P_i = p_{1,i}+2p_{2,i}$, we have:

$$p_{1,i}^O = \frac{3b_i+2\delta_i p_{2,i}}{2(3+2\delta_i)}. \tag{A.26}$$

Oligopoly Price for the High-cost Firm

Taking the low-cost firm's price $p_{1,i}$ choice as given and considering $k_{2,i}=k_{3,i}=k$, the profit maximization for each high-cost firm is given as follows:

$$\pi_{2,i}^O = \frac{1}{3}(p_{2,i}-k)[3b_i+\delta_i P_{-2,i}-p_{2,i}(3+2\delta_i)].$$

The fist order condition yield:

$$\frac{\partial\pi_{2,i}^O}{\partial p_{2,i}} = 0 \rightarrow b_i+\frac{\delta_i}{3}(p_{1,i}+p_{3,i})-\frac{2(3+2\delta_i)}{3}p_{2,i}+\frac{(3+2\delta_i)}{3}k=0.$$

$$3b_i+\delta_i P_{-2,i}+(3+2\delta_i)k=2p_{2,i}(3+2\delta_i).$$

Adding and subtracting $\delta_i p_{2,i}$:

$$3b_i + \delta_i P_i + (3+2\delta_i)k = p_{2,i}(6+5\delta_i). \tag{A.27}$$

Setting $P_i = p_{1,i} + 2p_{2,i}$, we have:

$$p^O_{2,i} = \frac{3b_i + \delta_i p_{1,i} + (3+2\delta_i)k}{3(2+\delta_i)}. \tag{A.28}$$

Equations (A.26) and (A.28) reveals that the price charged by the low-cost type firm is strictly lower than the price charged by the high-cost type firm. Solving the implied expression for both types of firms we have:

$$p^O_{1,i} = \frac{3(6+5\delta_i)b_i + 2(3+2\delta_i)\delta_i k}{6(3+2\delta_i)(2+\delta_i) - 2\delta_i^2}; \quad p^O_{2,i} = \frac{3(6+5\delta_i)b_i + 2(3+2\delta_i)^2 k}{6(3+2\delta_i)(2+\delta_i) - 2\delta_i^2}. \tag{A.29}$$

To calculate the average price, $\bar{p}$, we proceed as follows:

$$\bar{p} = \frac{1}{3}p^O_{1,i} + \frac{2}{3}p^O_{2,i}. \tag{A.30}$$

In order to find $q_1.q_2$ we combine equations (3.1), (A.29) and (A.30). Thus,

$$p^O_{1,i} - \bar{p} = -\frac{2}{3}\frac{(3+2\delta_i)k}{(6+5\delta_i)}. \tag{A.31}$$

Doing the same procedure for p_2:

$$p^O_{2,i} - \bar{p} = \frac{1}{3}\frac{(3+2\delta_i)k}{(6+5\delta_i}. \tag{A.32}$$

Now we can write the demand function as follows:

$$q^O_{1,i}(p_{1,i},\bar{p}) = \frac{[3(6+5\delta_i)b_i + 2\delta_i(3+2\delta_i)k](3+2\delta_i)}{6(6+5\delta_i)(3+\delta_i)}, \tag{A.33}$$

$$q^O_{2,i}(p_{2,i},\bar{p}) = \frac{[3(6+5\delta_i)b_i - 2(9+9\delta_i+\delta_i^2)k](3+2\delta_i)}{6[3(6+7\delta_i)+5\delta_i^2]}. \tag{A.34}$$

By combining prices and quantities, we compute the following profits:

$$\pi^O_{1,i} = \frac{1}{12}\frac{[3(6+5\delta_i)b_i + 2\delta_i(3+2\delta_i)k]^2(3+2\delta_i)}{(6+5\delta_i)^2(3+\delta_i)^2}, \tag{A.35}$$

$$\pi^O_{2,i} = \frac{1}{12}\frac{[3(6+5\delta_i)b_i - 2(9+9\delta_i+\delta_i^2)k]^2(3+2\delta_i)}{[3(6+7\delta_i)+5\delta_i^2]^2}. \tag{A.36}$$

Cartel Prices for the Heterogeneous Firms

The F.O.C. derived from equation (3.7) yields:

$$p_{1,i}^M = \frac{3b_i + 2\delta_i(2p_{2,i} - k)}{2(3+2\delta_i)}; \quad p_{2,i}^M = \frac{3b_i + (3+\delta_i)k + 2\delta_i p_{1,i}}{2(3+\delta_i)}.$$

Solving for $p_{1,i}^M$ and $p_{2,i}^M$:

$$p_{1,i}^M = \frac{1}{2}b_i; \quad p_{2,i}^M = \frac{1}{2}(b_i + k). \tag{A.37}$$

Equation (A.37) imply that $p_{2,i}^M > p_{1,i}^M$. By comparing equations (A.29) and (A.37) and holding δ_i, b_i and k constants, it is true that $p_{1,i}^M > p_{1,i}^O$ and $p_{2,i}^M > p_{2,i}^O$. As the regulator has sufficient technology to distinguish low-cost and high-cost firms, rather than adopting $p_{2,i}^M$ as the cartel price, firms will maximize their payoffs at their own prices. The motivation behind adopting different prices is to make regulation more difficult. Therefore, combining (3.1) and (A.37) the quantities produced by each type of firm is:

$$q_{1,i}^M = \frac{1}{2}b_i + \frac{1}{3}\delta_i k; \quad q_{2,i}^M = \frac{1}{2}b_i - \frac{(3+\delta_i)}{6}k. \tag{A.38}$$

The payoffs for the low and high-cost are given by equations (3.35) and (3.36), respectively.

Deviation and Retaliation in the Cartel with Heterogeneous Firms

Firms 1 and 2 set the cartel prices derived in equation (A.37). By it turn, firm 3 takes (A.37) into account and then chooses non-negative $p_{3,i}$ to maximize (3.37). Where $P_{-3,i} = b_i + \frac{k}{2}$. The first order condition yields:

$$p_{3,i} = \frac{2(3+\delta_i)b_i + (6+5\delta_i)k}{4(3+2\delta_i)}. \tag{A.39}$$

As the payoff (3.37) reveals,

$$q_{3,i} = \frac{3+\delta_i}{6}b_i - \frac{(2+\delta_i)}{4}k. \tag{A.40}$$

Thus, we can compute the Firm 3's payoff as in equation (3.38). For the firms in the cartel we derive the payoffs as follows. Since the quantity demanded from each cartel member equals:

$$\begin{aligned} q_{1,i}^M &= \frac{1}{3}[3b_i + \delta_i(p_{2,i}^M + p_{3,i}) - p_{1,i}^M(3+2\delta_i)], \\ q_{2,i}^M &= \frac{1}{3}[3b_i + \delta_i(p_{1,i}^M + p_{3,i}) - p_{2,i}^M(3+2\delta_i)], \end{aligned} \tag{A.41}$$

combining equations (A.37), (A.39) and (A.41) yields:

$$q_{1,i}^M = \frac{2(9+6\delta_i - \delta_i^2)b_i + 3\delta_i(4+3\delta_i)k}{12(3+2\delta_i)}, \tag{A.42}$$

$$q_{2,i}^M = \frac{2(9+6\delta_i-\delta_i^2)b_i - 3(6+6\delta_i+\delta_i^2)k}{12(3+2\delta_i)}. \tag{A.43}$$

Given (A.42) and (A.43), each cartel member earns the expected payoff given by equation (3.39). As the cartel steals a quantity s from the defector, then the price and quantity of firm 3 during retaliation is given by the F.O.C. in equation 3.40, and can be deduced as follows:

$$p_{3,i} = \frac{3b_i + \delta_i P_{-3,i} - s}{2(3+2\delta_i)} + \frac{k}{2}. \tag{A.44}$$

Given (A.44) the quantity sold by firm 3 in the retaliation period is:

$$q_{3,i} = \frac{3b_i + \delta_i P_{-3,i} - s - (3+2\delta_i)k}{6}. \tag{A.45}$$

Besides, the constraint in (3.41) follows from (A.45) and from the fact that the quantity sold by firm 3 cannot be negative. Maximizing equation (3.44), the F.O.C yields:

$$p_{1,i}^M = \frac{3b_i - 2\delta_i c}{4}; \qquad p_{2,i}^M = \frac{3b_i - 2\delta_i c}{4} + \frac{k}{2}. \tag{A.46}$$

Given (A.46), we have:

$$\begin{aligned} q_{1,i}^M &= 3b_i + \delta_i p_{2,i}^M - p_{1,i}^M(2+\delta_i) = \frac{3b_i + 2\delta_i c}{2} + \frac{\delta_i}{2}k; \\ q_{2,i}^M &= 3b_i + \delta_i p_{1,i}^M - p_{2,i}^M(2+\delta_i) = \frac{3b_i + 2\delta_i c}{2} - \frac{(2+\delta_i)}{2}k. \end{aligned} \tag{A.47}$$

Now, we can combine the inequality in (3.42) with the prices found in (A.46) to obtain the inequality in (3.46). Equations (A.46) and (A.47) enable us to calculate each cartel member's expected payoff as in equation (3.47). If inequality (3.43) holds, the F.O.C derived from expression (3.49) yields the following prices:

$$\begin{aligned} p_{1,i}^M &= \frac{1}{8}\left\{\frac{2[3(6+5\delta_i)+\delta_i^2]b_i + \delta_i(6+5\delta_i)k}{9+9\delta_i+2\delta_i^2}\right\}; \\ p_{2,i}^M &= \frac{1}{8}\left\{\frac{2[3(6+5\delta_i)+\delta_i^2]b_i + [6(6+7\delta_i)+13\delta_i^2]k}{9+9\delta_i+2\delta_i^2}\right\}. \end{aligned} \tag{A.48}$$

Combining (A.39) and (A.48), we have the following quantities:

$$\begin{aligned} q_{1,i}^M &= \frac{1}{24}\left\{\frac{2[3(6+5\delta_i)+\delta_i^2]b_i + \delta_i(18+13\delta_i)k}{3+2\delta_i}\right\}; \\ q_{2,i}^M &= \frac{1}{24}\left\{\frac{2[3(6+5\delta_i)+\delta_i^2]b_i - [6(6+7\delta_i)+11\delta_i^2]k}{3+2\delta_i}\right\}. \end{aligned} \tag{A.49}$$

Combining (A.48) and (A.49) we can find the expected payoff earned by each cartel member as in expressions (3.50) and (3.51). In sequence, combining equations (A.44) and (A.48) yields:

$$p_{3,i} = \frac{2[\delta_i^3 + 9(6+8\delta_i+3\delta_i^2)]b_i + [25\delta_i^3 + 6(18+33\delta_i+20\delta_i^2)]k}{8(3+2\delta_i)^2(3+\delta_i)}. \tag{A.50}$$

Combining equations (A.45) and (A.48) yields:

$$q_{3,i} = \frac{2[\delta_i^3 + 9(6 + 8\delta_i + 3\delta_i^2)]b_i - [7\delta_i^3 + 18(6 + 9\delta_i + 4\delta_i^2)]k}{24(9 + 9\delta_i + 2\delta_i^2)}. \tag{A.51}$$

Given equations (A.50) and (A.51), we can compute firm 3's expected payoff as in expression (3.52).

N Heterogeneous Firms

We start assuming the case in which a high-cost firm deviates from the cartel. Maximizing (3.57) with respect to prices $\{p_{l,i}, p_{h,i}\}$ yields to:

$$p_{l,i} = \frac{Nb_i + N_{h,i}\delta_i(2p_{h,i} - k)}{2[N + (N - N_{l,i})\delta_i]}; \qquad p_{h,i} = \frac{Nb_i + [N + (N - N_{h,i})\delta_i]k + 2p_{l,i}N_{l,i}\delta_i}{2[N + (N - N_{h,i})\delta_i]}.$$

When a high-cost firm deviates, the F.O,C in equation (3.60) yields:

$$p_{h,i}^D = \frac{[2N + (N_{l,i} - N_{h,i})\delta_i + \delta_i]b_i + [2N(1 + \delta_i) - (N_{h,i} + 1)\delta_i]k}{4[N + (N - 1)\delta_i]}. \tag{A.52}$$

As the payoff (3.60) reveals,

$$q_{h,i}^D = \frac{1}{N}\Big\{Nb_i + P_{-h}^M\delta_i - p_{h,i}^D[N + (N - 1)\delta_i]\Big\}. \tag{A.53}$$

Combining equations (A.52) and (A.53) yields:

$$q_{h,i}^D = \frac{[2N + (N_{l,i} - N_{h,i})\delta_i + \delta_i]b_i - [2N(1 + \delta_i) + (N_{h,i} - 3)\delta_i]k}{4N}. \tag{A.54}$$

Thus, we can find firm $\pi_{h,i}^D$'s expected payoff in the first period as in equation (3.61). Now, we consider the case in which a low-cost firm deviates from the collusion. The F.O.C derived from equation (3.67) yields:

$$p_{l,i}^D = \frac{[2N + (N_{h,i} - N_{l,i} + 1)\delta_i]b_i + N_{h,i}\delta_i k}{4[N + (N - 1)\delta_i]}. \tag{A.55}$$

As the payoff (3.67) reveals,

$$q_{l,i}^D = \frac{1}{N}\Big\{Nb_i + P_{-l}^M\delta_i - p_{l,i}^D[N + (N - 1)\delta_i]\Big\}. \tag{A.56}$$

Combining equations (A.55) and (A.56) yields

$$q_{l,i}^D = \frac{[2N + (N_{h,i} - N_{l,i} + 1)\delta_i]b_i + N_{h,i}\delta_i k}{4N}. \tag{A.57}$$

Thus, we can find firm $\pi_{l,i}^D$'s expected payoff in the first period as in equation (3.68).